CHAPTER

4

# Gravity extends throughout the universe.

### Key Concepts

In this chapter, you will learn about:

- gravitational force
- Newton's law of universal gravitation
- gravitational field

### Learning Outcomes

When you have completed this chapter, you will be able to:

**Knowledge**

- identify gravity as a fundamental force in nature
- describe Newton's law of universal gravitation
- explain the Cavendish experiment
- define and apply the concept of a gravitational field
- compare gravitational field strength and acceleration due to gravity
- predict the weight of objects on different planets

**Science, Technology, and Society**

- explain that concepts, models, and theories help interpret observations and make predictions

Skydiving, hang-gliding, bungee jumping, and hot-air ballooning are just a few activities that involve accelerating under the influence of gravity for a thrill (Figure 4.1). Gravitational force attracts all objects in the universe. It holds you to Earth, and Earth in its orbit around the Sun.

In 1665, Isaac Newton began his study of gravity when he attempted to understand why the Moon orbits Earth. His theories led him to an understanding of the motion of planets and their moons in the solar system. Several centuries later, these theories led to the launch of satellites and the success of various space missions such as Mariner and Voyager.

Gravity is one of the four basic forces of nature, called **fundamental forces**, that physicists think underlie all interactions in the universe. These forces are the gravitational force, the electromagnetic force, the weak nuclear force, and the strong nuclear force.

In this chapter, you will investigate how gravity affects the motion of objects on Earth and on other planets, and how it affects the motion of satellites orbiting Earth.

▲ **Figure 4.1** Understanding how forces and gravity affect motion determines how successful the design of a hot-air balloon will be and the best way to navigate it.

## 4–1 QuickLab

# Falling Coins

### Problem

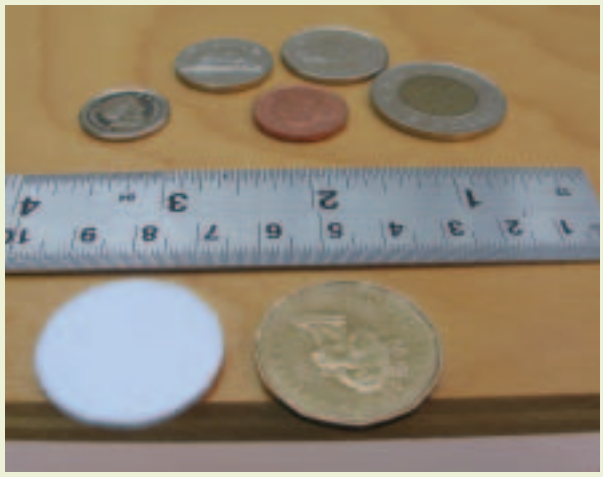
Suppose you drop two coins of different shapes and sizes from the same height. How do the rates of both coins falling compare?

### Materials

variety of coins (penny, nickel, dime, quarter, loonie, and toonie)
ruler
Styrofoam™ disk (size of a loonie)

### Procedure

1. Choose any two different coins and place them at the edge of a table above an uncarpeted floor.
2. Using a ruler, push the coins off the table so they leave at the same time (Figure 4.2).
3. Listen carefully for the sounds of the coins as they hit the floor.
4. Repeat this activity with different combinations of two coins, including the loonie with the Styrofoam™ disk. Record your observations.



▲ **Figure 4.2**

### Questions

1. When the coins landed, how many sounds did you hear? Did all combinations of two coins give the same result? Explain.
2. If all the coins fall at the same rate, how many sounds would you expect to hear when they land?
3. How would the results compare if two coins were released at the same time from a greater height, such as 10 m?
4. How did the average acceleration of the loonie differ from that of the Styrofoam™ disk? Explain why.

## Think About It

1. (a) What factors affect the weight of an astronaut during a rocket flight?

   (b) How does the astronaut's weight change?

2. What would be the motion of Earth if the Sun's gravity were zero? Assume that no other celestial bodies affect Earth.

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes to your ideas.

# 4.1 Gravitational Forces due to Earth

*info* **BIT**

Gravitational force is an attraction force only. There is no such thing as a repulsive gravitational force.

**gravitational force:** attractive force between any two objects due to their masses

One of Newton's great achievements was to identify the force that causes objects to fall near Earth's surface as the same force that causes the Moon to orbit Earth. He called this force "gravity," and he reasoned that this force is present throughout the universe.

**Gravitational force**, $\vec{F}_g$, is the force that attracts any two objects together. Although this force is the weakest fundamental force, you can feel its effect when you interact with an object of very large mass such as Earth.

When you slide down a waterslide, you can feel the gravitational force exerted by Earth pulling you downward toward the bottom of the slide (Figure 4.3). But if you want to feel the gravitational force exerted by the person sitting next to you, you will not be able to sense anything because the magnitude of the force is so small.

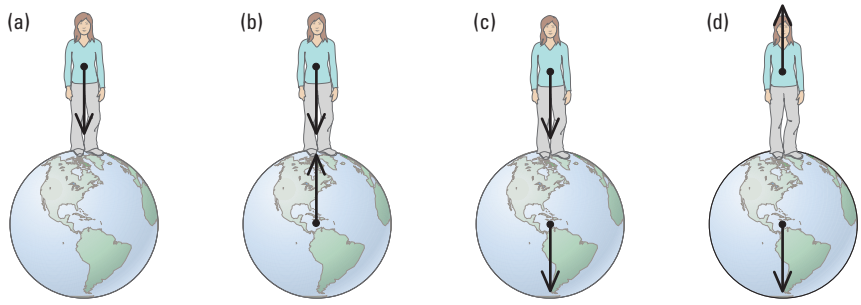
▶ **Figure 4.3** The attractive force between Earth and you is far greater than that between you and another person coming down a waterslide.

Gravitational force is a force that always exists in pairs. This is another example of Newton's third law. If Earth exerts a gravitational force of magnitude 700 N on you, then you exert a gravitational force of magnitude 700 N on Earth. Earth attracts you and you attract Earth.

The force you exert on Earth has a negligible effect because Earth's mass is huge in comparison to yours ($5.97 \times 10^{24}$ kg). However, the gravitational force that Earth exerts on you causes a noticeable acceleration because of your relatively small mass.

### Concept Check

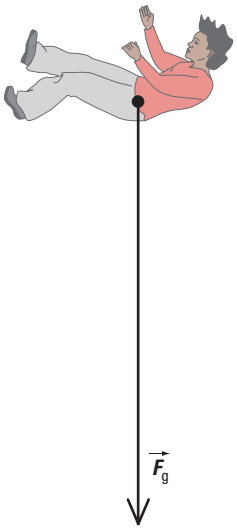
Which diagram best represents the gravitational force acting on you and on Earth (Figure 4.4)? Explain your reasoning.



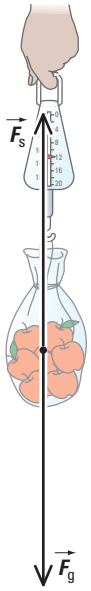

▲ **Figure 4.4**

## The Concept of Weight

In a vacuum, all objects near Earth's surface will fall with the same acceleration, no matter what the objects consist of or what their masses are. The only force acting on a falling object in a vacuum is the gravitational force exerted by Earth on the object (Figure 4.5). Suppose you analyze this situation using a free-body diagram and Newton's second law.




▲ **Figure 4.5** Free-body diagram for a falling object in a vacuum




▲ **Figure 4.6** Diagram showing the forces acting on an object that is suspended from a spring scale

**weight:** gravitational force exerted on an object by a celestial body

The equation for the net force acting on the falling object is

$$\vec{F}_{net} = \vec{F}_g$$

$$m\vec{a} = \vec{F}_g$$

Since the object is accelerating due to gravity, $\vec{a} = \vec{g}$. So the equation for the net force becomes

$$m\vec{g} = \vec{F}_g$$

or

$$\vec{F}_g = m\vec{g}$$

The equation $\vec{F}_g = m\vec{g}$ is valid in general, because the gravitational force acting on an object is the same, whether or not the object is at rest or is moving. This equation relates the gravitational force acting on an object, the so-called **weight** of the object, to its mass.

One way to measure the magnitude of the weight of an object directly involves using a spring scale (Figure 4.6). When the object stops moving at the end of the spring, Earth exerts a downward gravitational force on the object while the spring exerts an upward elastic force of equal magnitude on the object.

Find out what the relationship is between mass and gravitational force in the vicinity of your school by doing 4-2 QuickLab.

## 4-2 QuickLab

# Relating Mass and Weight

### Problem

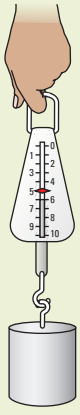
What is the relationship between the mass of an object and the local value of the gravitational force exerted on that object?

### Materials

set of standard masses with hooks
spring scale (0–10 N)
graph paper

### Procedure

1. Design a procedure to determine the gravitational force acting on a set of standard masses (Figure 4.7).
2. Use a table to record the magnitude of the gravitational force and mass. Add another column in the table for the ratio of $F_g$ to $m$.
3. Calculate the ratio of $F_g$ to $m$ for each standard mass. Calculate the average ratio for all masses. Include units.



▲ **Figure 4.7**

4. Plot a graph of $F_g$ vs. $m$. Draw a line of best fit through the data points. Calculate the slope of the graph.

### Questions

1. What does the ratio of $F_g$ to $m$ represent? How constant is this value?
2. Describe the graph of $F_g$ vs. $m$. How does the slope compare to the average ratio calculated in step 3?
3. Write an equation relating $F_g$ and $m$. Use the symbol $g$ for the proportionality constant.
4. (a) The Moon exerts a gravitational force that is about $\frac{1}{6}$ that exerted by Earth. If you did this activity on the Moon using an appropriate spring scale and the same standard masses, would
   - the graph be a straight line?
   - the slope be the same as before?
   - the line go through the origin?
   - the proportionality constant $g$ be the same as before?

   (b) Why would a 0–5 N spring scale be more ideal to use on the Moon, rather than a 0–10 N spring scale?

## Gravitational Mass

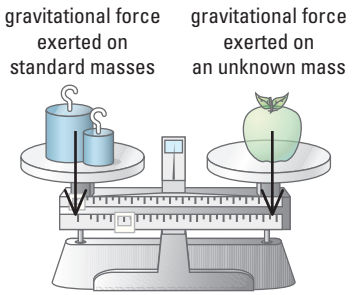
The mass in the equation $\vec{F}_g = m\vec{g}$ is determined by finding the ratio of the gravitational force acting on an object to the acceleration due to gravity. Since $\vec{F}_g$ and $\vec{g}$ are in the same direction, the scalar form of the equation may be used:

$$F_g = mg$$

$$m = \frac{F_g}{g}$$

A practical way to measure this mass involves using a balance. In Figure 4.8, an object of unknown mass (A) is placed on one pan and standard masses (B) are added to the other pan until both pans balance. This method involves comparing the weights of two objects: one unknown and the other known. Mass measured using the concept of weight is called **gravitational mass**.

**gravitational mass:** mass measurement based on comparing the known weight of one object to the unknown weight of another object




◀ **Figure 4.8** When both pans are balanced, the gravitational force acting on the unknown mass is equal to the gravitational force acting on the standard masses.

If the balance in Figure 4.8 were moved to the Moon, the process of determining the gravitational mass of object A would be the same. However, the weight of A and B would be different from that at Earth's surface because the acceleration due to gravity at the Moon's surface, $g_{Moon}$, is 1.62 m/s$^2$ compared to 9.81 m/s$^2$ at Earth's surface.

$$F_{g_A} = F_{g_B}$$

$$m_A g_{Moon} = m_B g_{Moon}$$

$$m_A = m_B$$

But since both objects A and B experience the same value of $g_{Moon}$, $m_A = m_B$. So the gravitational mass of an object is the same whether the object is on Earth, the Moon, or anywhere else in the universe. Both gravitational mass and inertial mass are properties of an object that do not depend on the location of the object.

**eSIM**

Explore how mass and weight are measured. Follow the eSim links at www.pearsoned.ca/school/physicssource.

## Is Inertial Mass the Same as Gravitational Mass?

You can determine the mass of an object by using either the concept of inertia or weight. Experiments since Newton's day have shown that for any object, the numerical value of its inertial mass is equal to its gravitational mass. Later, Albert Einstein (1879–1955) showed that inertial mass is actually equivalent to gravitational mass. So it does not matter whether you determine the mass of an object using inertia or weight, because the numerical value of the mass is the same.

## Describing Gravitational Force as a Field

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Physicists are attempting to find a way to view all fundamental forces as different aspects of a single concept. So far, physicists have successfully unified the electromagnetic and the weak nuclear forces into the electroweak interaction.

Gravitational force is an example of a force that acts on objects whether or not they actually touch each other, even if the objects are in a vacuum. These forces are referred to as **action-at-a-distance forces**.

Since Newton's time, physicists have introduced the concept of a field to explain action-at-a-distance forces. You encountered some fields in previous science courses when you worked with magnets.

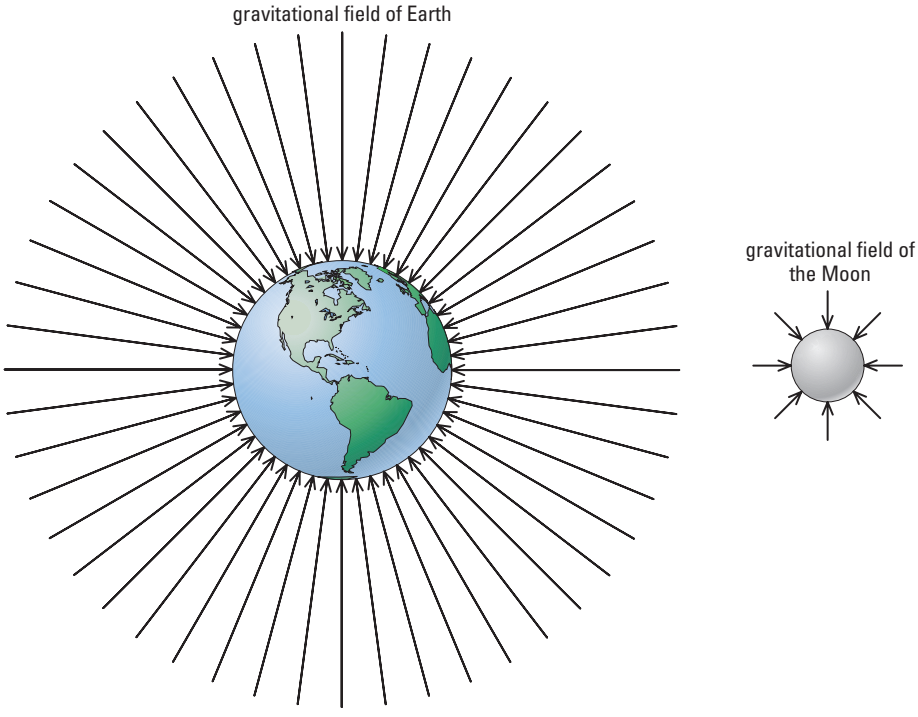
Imagine you are moving the north pole of a magnet close to the north pole of a fixed magnet. As you move the magnet closer to the fixed magnet, you can feel an increasing resistance. Somehow, the fixed magnet has created a region of influence in the space surrounding it. Physicists refer to a three-dimensional region where there is some type of an influence, whether it is an attraction or a repulsion, on a suitable object as a **field**.

Since every object exerts a gravitational force in three dimensions, it influences the space around it (Figure 4.9). This region of influence is a **gravitational field**, and it is through this region that two objects interact.

**action-at-a-distance force:** force that acts even if the objects involved are not touching

**field:** three-dimensional region of influence

**gravitational field:** region of influence surrounding any object that has mass




▲ **Figure 4.9** This figure shows a two-dimensional representation of Earth's and the Moon's gravitational field. A gravitational field is three-dimensional and is directed toward the centre of the object.

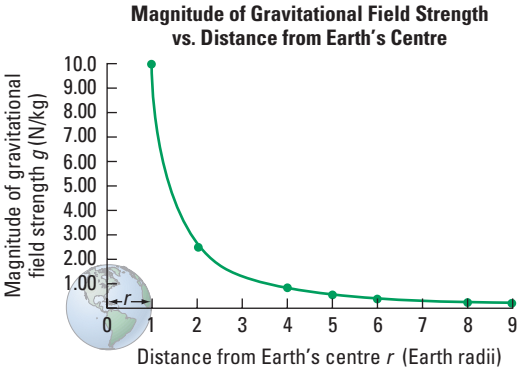
To determine the magnitude and direction of a gravitational field created by an object, you could use a test mass $m_{test}$. At different locations around the object, this test mass will experience a gravitational force that has a certain magnitude and direction. The direction of the gravitational force will be directed toward the centre of the object.

**Gravitational field strength** is defined as the gravitational force per unit mass, $\vec{g} = \frac{\vec{F}_g}{m_{test}}$. If you release the test mass, it will accelerate toward the object with an acceleration equal to $g$.

**gravitational field strength:** gravitational force per unit mass at a specific location

Figure 4.10 shows how the magnitude of Earth's gravitational field strength changes as a test mass is moved farther away from Earth's centre. The farther the test mass is moved, the more significant is the decrease in $g$. In fact, the graph in Figure 4.10 shows an inverse square relationship:

$$g \propto \frac{1}{r^2}$$




▲ **Figure 4.10** The magnitude of the gravitational field strength as a function of distance from Earth's centre

Since force is measured in newtons and mass in kilograms, the units of gravitational field strength are newtons per kilogram, or N/kg. The ratio you determined in 4-2 QuickLab was the gravitational field strength at the vicinity of your school.

## Concept Check

What happens to the magnitude of the gravitational field strength if
(a) $r$ decreases by a factor of four?
(b) $r$ increases by a factor of two?
(c) $m_{test}$ doubles?
(d) $m_{test}$ is halved?

## 4.1 Check and Reflect

### Knowledge

1. Distinguish between mass and weight. Explain using an example and a diagram.
2. Distinguish between inertial mass and gravitational mass.
3. In your own words, define gravitational acceleration and gravitational field strength. State the units and symbol for each quantity.
4. In your own words, explain the concept of a gravitational field. Include an example of how a gravitational field affects another object.
5. Why do physicists use the concept of a field to describe gravity?
6. In a vacuum, a feather and a bowling ball are released from rest at the same time from the same height. Compare the time it takes for each object to fall. Explain your answer.

### Applications

7. The Moon exerts a gravitational force that is about $\frac{1}{6}$ that exerted by Earth. Explain why the mass of an object measured on the Moon using a balance is the same as if the object were on Earth's surface.
8. Describe a situation where measuring the inertial mass of an object is easier than measuring its gravitational mass.
9. The table below shows the magnitude of the gravitational force on objects of different mass in Banff, Alberta.

| Mass (kg) | 0 | 1.50 | 3.00 | 4.50 | 6.00 | 7.50 | 10.0 |
|---|---|---|---|---|---|---|---|
| Magnitude of Gravitational Force (N) | 0 | 14.7 | 29.4 | 44.1 | 58.9 | 73.6 | 98.1 |

   (a) Graph the data.
   (b) Calculate the slope of the line.
   (c) What does the slope represent?

10. How could you distinguish between a 5.0-kg medicine ball and a basketball in outer space without looking at both objects?

### Extensions

11. Visit a local fitness gymnasium. Find out how athletes use gravitational and elastic forces to improve their fitness. Is friction a help or a hindrance? Write a brief report of your findings.
12. List some occupations that might require a knowledge of gravitational field strength. Briefly explain how gravitational field strength applies to these occupations.
13. Complete the gathering grid below to summarize the similarities and differences among gravitational mass, inertial mass, and gravitational force.

| | Gravitational Mass | Inertial Mass | Gravitational Force |
|---|---|---|---|
| Definition | | | |
| SI unit | | | |
| Measuring Instrument(s) | | | |
| How the Quantity Is Measured | | | |
| Factors It Depends On | | | |
| Variability with Location | | | |

**eTEST**

To check your understanding of gravitational force, weight, mass, and gravitational field strength, follow the eTest links at www.pearsoned.ca/school/physicssource.

# 4.2 Newton's Law of Universal Gravitation

Gravity affects all masses in the universe. No matter where you are on Earth or in outer space, you exert a gravitational force on an object and an object exerts a gravitational force, of equal magnitude but opposite direction, on you. Because gravitational force acts over any distance, the range of its effect is infinite.

Near Earth's surface, the magnitude of the gravitational force exerted by Earth (object A) on object B is given by the equation $F_{\text{A on B}} = m_B g$. But object B also exerts a gravitational force of equal magnitude on Earth, $F_{\text{B on A}} = m_A g$. Newton hypothesized that, given two objects A and B, the magnitude of the gravitational force exerted by one object on the other is directly proportional to the product of both masses:

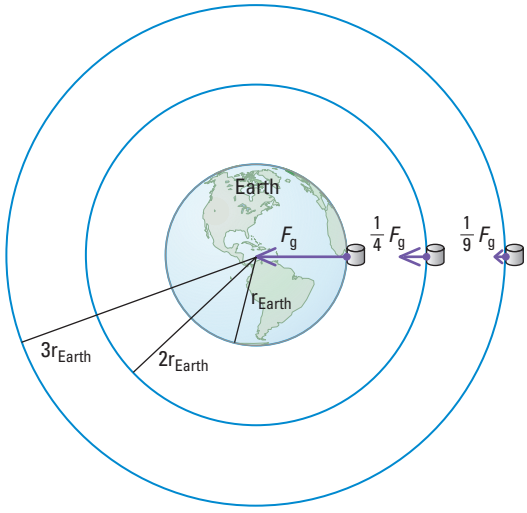
$$F_g \propto m_A m_B$$

Figure 4.11 shows the magnitude of the gravitational force acting on an object at Earth's surface ($r_{\text{Earth}}$), one Earth radius above Earth ($2r_{\text{Earth}}$), and two Earth radii above Earth ($3r_{\text{Earth}}$). If the separation distance from Earth's centre to the centre of the object doubles, $F_g$ decreases to $\frac{1}{4}$ of its original value. If the separation distance from Earth's centre to the centre of the object triples, $F_g$ decreases to $\frac{1}{9}$ of its original value. So, just as $g \propto \frac{1}{r^2}$, $F_g$ is inversely proportional to the square of the separation distance (Figure 4.12):

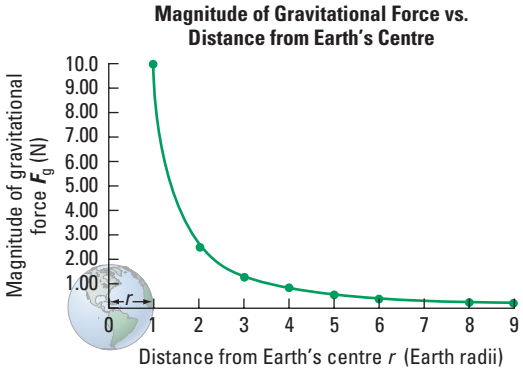
$$F_g \propto \frac{1}{r^2}$$

*info* **BIT**

Gravity is the dominant force throughout the universe. This force is attractive and has an infinite range.




▲ **Figure 4.11** The magnitude of the gravitational force acting on an object some distance from Earth varies inversely with the square of the separation distance.




▲ **Figure 4.12** The magnitude of the gravitational force acting on a 1.00-kg object as a function of distance from Earth's centre

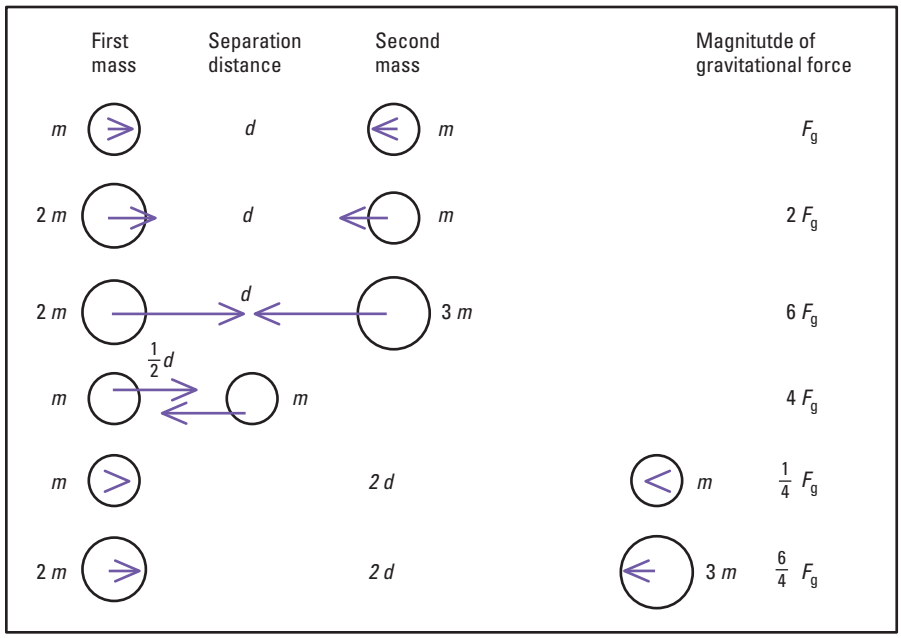
If you combine both proportionalities into one statement, you get $F_g \propto \frac{m_A m_B}{r^2}$ or $F_g = \frac{G m_A m_B}{r^2}$ (Figure 4.13). This mathematical relationship is Newton's law of universal gravitation.

**eSIM**

Explore the relationship among $m_A$, $m_B$, $r$, and $F_g$ in Newton's law of gravitation. Follow the eSim links at www.pearsoned.ca/school/physicssource.

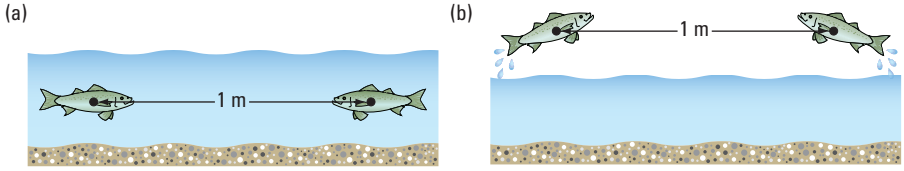
Any two objects, A and B, in the universe exert gravitational forces of equal magnitude but opposite direction on each other. The forces are directed along the line joining the centres of both objects. The magnitude of the gravitational force is given by $F_g = \frac{G m_A m_B}{r^2}$,

where $m_A$ and $m_B$ are the masses of the two objects, $r$ is the separation distance between the centres of both objects, and $G$ is a constant called the **universal gravitational constant**.




▲ **Figure 4.13** The magnitude of the gravitational force is directly proportional to the product of the two masses, and inversely proportional to the square of the separation distance.

Experiments have shown that the magnitude of the gravitational force acting on any pair of objects does not depend on the medium in which the objects are located (Figure 4.14). In other words, given two fish, the gravitational force acting on either fish will have the same magnitude if both fish are underwater or in midair.




▲ **Figure 4.14** The magnitude of the gravitational force acting on either fish is the same whether both fish are (a) underwater or (b) above water.

## Concept Check

Two identical stationary baseballs are separated by a distance $r$. What will happen to the magnitude of the gravitational force acting on either ball if

(a) the mass of each ball doubles?

(b) $r$ is halved?

(c) the mass of each ball is halved and $r$ doubles?

**e WEB**

Edmund Halley, an associate of Newton, paid for the publication of some of Newton's famous work. Find out about Edmund Halley and his contributions to Newton's work. Begin your search at www.pearsoned.ca/school/physicssource.

## Determining the Value of the Universal Gravitational Constant

Although Newton found a mathematical relationship for gravitational force, he was unable to determine the value of $G$. In 1798, scientist Henry Cavendish (1731–1810) confirmed experimentally that Newton's law of gravitation is valid, and determined the density of Earth. Cavendish's experimental set-up was later used to determine the value of $G$.

The magnitude of the gravitational force acting on most pairs of objects is very weak and the magnitude decreases significantly as the separation distance between the objects increases. However, if you use two light spheres (each of mass $m$) and two heavy spheres (each of mass $M$) that are very close to each other, it is possible to determine the magnitude of the gravitational force exerted by $M$ on $m$.

The trick is to use a device that can accurately measure the very small gravitational force. A modern **torsion balance** is a device that uses a sensitive fibre and a beam of light to measure very minute forces due to gravity, magnetic fields, or electric charges. Cavendish used a modified torsion balance invented by John Michell (1724–1793) to verify Newton's law of gravitation.

**torsion balance:** device used to measure very small forces

A modern torsion balance consists of a small, light, rigid rod with two identical, light, spheres ($m$) attached to each end (Figure 4.15). The rod is suspended horizontally by a thin fibre connected to the centre of the rod. A mirror is also attached to the fibre and rod so that when the rod turns, the mirror also turns by the same amount. The entire assembly is supported in an airtight chamber. The torsion balance initially experiences no net force and the spheres $m$ are stationary.

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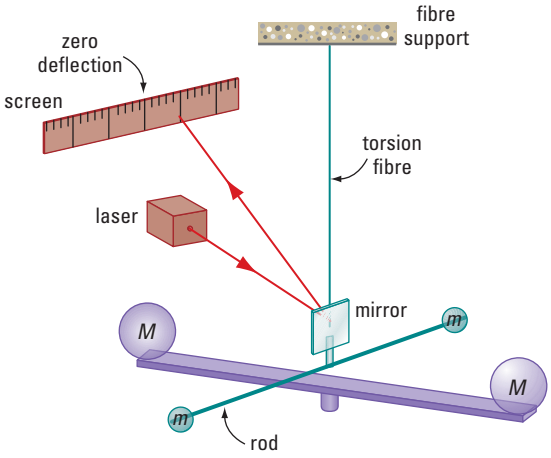
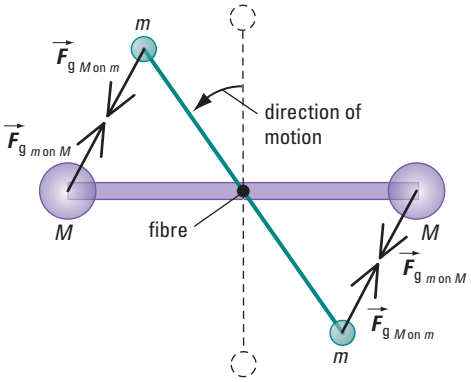
Charles de Coulomb invented the torsion balance in 1777 to measure small magnetic forces and forces in fluids. However, John Michell independently invented the same type of device in 1784.




◀ **Figure 4.15** A modern torsion balance uses a laser beam to measure the amount of twist in the fibre. The most accurate value of $G$ has been determined using such a device.




▲ **Figure 4.16** This figure shows the top view of spheres *m* and *M* from Figure 4.15. The original position of spheres *m* is shown with dashed lines. The gravitational force exerted by *M* on *m* causes *m* to rotate toward *M*. The greater the magnitude of the gravitational force, the greater the angle of rotation.

When two identical, heavy, spheres (*M*) are moved close to spheres *m*, the gravitational force exerted by *M* on *m* causes *m* to rotate horizontally toward *M*. This rotation causes the fibre to twist slightly (Figure 4.16). As the fibre twists, the mirror attached to both the fibre and the rod turns through an angle in the horizontal plane. A beam of light reflected from the mirror becomes deflected as spheres *m* rotate. The amount of deflection is an indication of how much the spheres rotate. The greater the magnitude of the gravitational force, the more the fibre twists, and the greater the angle of rotation.

By measuring the amount of deflection, the gravitational force exerted by *M* on *m* can be determined. Spheres *M* are then moved to a symmetrical position on the opposite side of *m*, and the procedure is repeated.

Since the separation distance between *m* and *M*, the values of *m* and *M*, and the gravitational force can all be measured, it is possible to calculate *G* using Newton's law of gravitation.

$$F_g = \frac{GmM}{r^2}$$

$$\frac{F_g r^2}{mM} = G$$

$$G = \frac{F_g r^2}{mM}$$

***info* BIT**

Scientists use Newton's law of universal gravitation to calculate the masses of planets and stars.

The current accepted value of *G* to three significant digits is $6.67 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$.

In Example 4.1, Newton's law of gravitation is used to show that a person weighs slightly less at the top of the mountain than at its base.

## Example 4.1

Mount Logan in the Yukon is 5959 m above sea level, and is the highest peak in Canada. Earth's mass is $5.97 \times 10^{24}$ kg and Earth's equatorial radius is $6.38 \times 10^6$ m. What would be the difference in the magnitude of the weight of a 55.0-kg person at the top of the mountain as compared to at its base (Figure 4.17)? Assume that Earth's equatorial radius is equal to the distance from Earth's centre to sea level.

***Given***

$m_p = 55.0$ kg

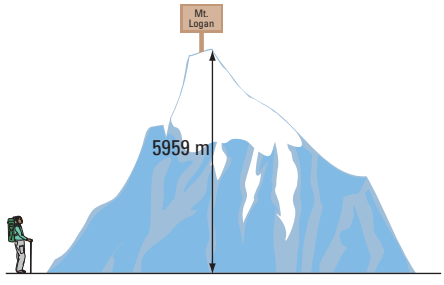
$h = 5959$ m

$m_{Earth} = 5.97 \times 10^{24}$ kg

$r_{Earth} = 6.38 \times 10^6$ m

***Required***

difference in magnitude of weight ($\Delta F_g$)




▲ **Figure 4.17**

***Analysis and Solution***

Assume that the separation distance between the person at the base of the mountain and Earth is equal to Earth's equatorial radius.

Base of mountain: $r_B = r_{Earth} = 6.38 \times 10^6 \text{ m}$

Top of mountain: $r_T = 6.38 \times 10^6 \text{ m} + 5959 \text{ m}$

The person's weight is equal to the gravitational force exerted by Earth on the person, and is directed toward Earth's centre both at the base and at the top of the mountain.

Calculate $F_g$ at the base of the mountain using Newton's law of gravitation.

$$(F_g)_B = \frac{Gm_p m_{Earth}}{(r_B)^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(55.0 \cancel{\text{kg}})(5.97 \times 10^{24} \cancel{\text{kg}})}{(6.38 \times 10^6 \text{ m})^2}$$

$$= 538.049 \text{ N}$$

Calculate $F_g$ at the top of the mountain using Newton's law of gravitation.

$$(F_g)_T = \frac{Gm_p m_{Earth}}{(r_T)^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\cancel{\text{kg}^2}}\right)(55.0 \cancel{\text{kg}})(5.97 \times 10^{24} \cancel{\text{kg}})}{(6.38 \times 10^6 \text{ m} + 5959 \text{ m})^2}$$

$$= 537.045 \text{ N}$$

The difference in the magnitude of the weight is equal to the difference in magnitude of both gravitational forces.

$$\Delta F_g = (F_g)_B - (F_g)_T$$
$$= 538.049 \text{ N} - 537.045 \text{ N}$$
$$= 1.00 \text{ N}$$

***Paraphrase***

The difference in the magnitude of the person's weight is 1.00 N.

## Practice Problems

1. Two people, A and B, are sitting on a bench 0.60 m apart. Person A has a mass of 55 kg and person B a mass of 80 kg. Calculate the magnitude of the gravitational force exerted by B on A.
2. The mass of the *Titanic* was $4.6 \times 10^7$ kg. Suppose the magnitude of the gravitational force exerted by the *Titanic* on the fatal iceberg was 61 N when the separation distance was 100 m. What was the mass of the iceberg?

**Answers**

1. $8.2 \times 10^{-7}$ N
2. $2.0 \times 10^8$ kg

## Using Proportionalities to Solve Gravitation Problems

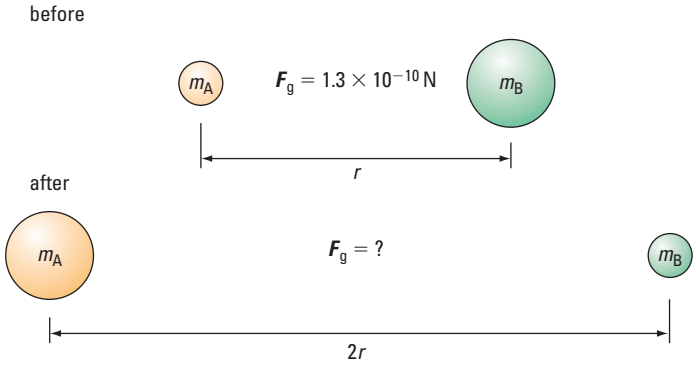
Example 4.2 demonstrates how to solve gravitation problems using proportionalities. This technique is useful if you are given how the separation distance and masses change from one situation to another.

## Example 4.2

Object A exerts a gravitational force of magnitude $1.3 \times 10^{-10}$ N on object B. Determine the magnitude of the gravitational force if the separation distance is doubled, $m_A$ increases by 6 times, and $m_B$ is halved. Explain your reasoning.

### *Analysis and Solution*

From Newton's law of gravitation, $F_g \propto m_A m_B$ and $F_g \propto \frac{1}{r^2}$. Figure 4.18 represents the situation of the problem.




▲ **Figure 4.18**

$$F_g \propto (6m_A)\left(\frac{1}{2}m_B\right) \qquad \text{and} \qquad F_g \propto \frac{1}{(2r)^2}$$

$$\propto (6)\left(\frac{1}{2}\right)m_A m_B \qquad \propto \left(\frac{1}{2^2}\right)\left(\frac{1}{r^2}\right)$$

$$\propto 3m_A m_B \qquad \propto \left(\frac{1}{4}\right)\left(\frac{1}{r^2}\right)$$

Calculate the factor change of $F_g$.

$$3 \times \frac{1}{4} = \frac{3}{4}$$

Calculate $F_g$.

$$\frac{3}{4}F_g = \frac{3}{4} \times (1.3 \times 10^{-10}\ \text{N})$$
$$= 1 \times 10^{-10}\ \text{N}$$

The new magnitude of the gravitational force will be $1 \times 10^{-10}$ N.

## Practice Problem

1. Object A exerts a gravitational force of magnitude $5.9 \times 10^{-11}$ N on object B. For each situation, determine the magnitude of the gravitational force. Explain your reasoning.
   (a) the separation distance increases to $\frac{4}{3}$ of its original value, $m_A$ increases to $\frac{3}{2}$ of its original value, and $m_B$ is halved
   (b) the separation distance decreases to $\frac{1}{6}$ of its original value, $m_A$ is halved, and $m_B$ increases to $\frac{5}{4}$ of its original value

**Answer**

1. (a) $2.5 \times 10^{-11}$ N
   (b) $1.3 \times 10^{-9}$ N

## Using Superposition to Find the Net Gravitational Force on an Object

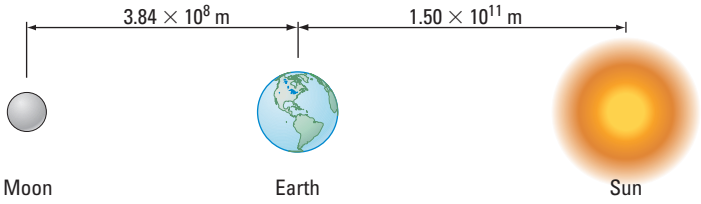
Example 4.3 demonstrates how to calculate the gravitational force exerted by both the Moon and the Sun on Earth. A free-body diagram is used to determine the gravitational forces acting on Earth. The technique of adding the gravitational force due to each pair of objects (Earth and the Moon, and Earth and the Sun) to find the net gravitational force is called superposition.

## Example 4.3

During a lunar eclipse, Earth, the Moon, and the Sun are aligned on the same plane as shown in Figure 4.19. Using the data in the chart below, calculate the net gravitational force exerted by both the Moon and the Sun on Earth.

| Celestial Body | Mass* (kg) | Mean Separation Distance from Earth* (m) |
|---|---|---|
| Earth | $5.97 \times 10^{24}$ | — |
| Earth's Moon | $7.35 \times 10^{22}$ | $3.84 \times 10^{8}$ |
| Sun | $1.99 \times 10^{30}$ | $1.50 \times 10^{11}$ |

*Source: Jet Propulsion Laboratory, California Institute of Technology (See JPL link at www.pearsoned.ca/school/physicssource.)




▲ **Figure 4.19**

### *Given*

$m_{Earth} = 5.97 \times 10^{24}$ kg

$m_{Moon} = 7.35 \times 10^{22}$ kg

$m_{Sun} = 1.99 \times 10^{30}$ kg

$r_{E\ to\ M} = 3.84 \times 10^{8}$ m

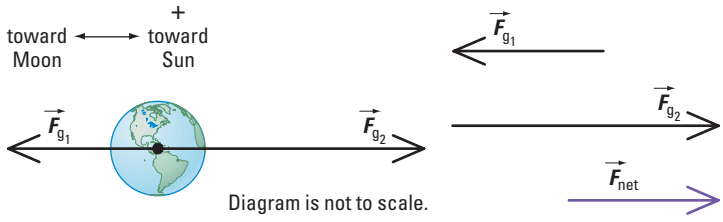
$r_{E\ to\ S} = 1.50 \times 10^{11}$ m

### *Required*

net gravitational force on Earth ($\vec{F}_g$)

### *Analysis and Solution*

Draw a free-body diagram for Earth (Figure 4.20).




▲ **Figure 4.20**

Calculate $F_g$ exerted by the Moon on Earth using Newton's law of gravitation.

$$(F_g)_1 = \frac{Gm_{Earth}m_{Moon}}{(r_{E\ to\ M})^2}$$

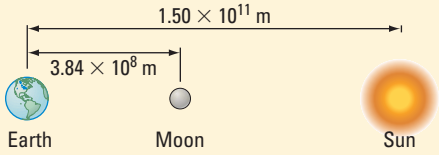
$$= \frac{\left(6.67 \times 10^{-11}\ \frac{N \cdot m^2}{\cancel{kg^2}}\right)(5.97 \times 10^{24}\ \cancel{kg})(7.35 \times 10^{22}\ \cancel{kg})}{(3.84 \times 10^{8}\ m)^2}$$

$$= 1.985 \times 10^{20}\ N$$

$\vec{F}_{g_1} = 1.985 \times 10^{20}$ N [toward Moon's centre]

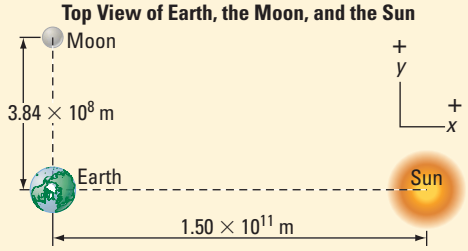
## Practice Problems

1. During a solar eclipse, Earth, the Moon, and the Sun are aligned on the same plane as shown in Figure 4.21. Calculate the net gravitational force exerted by both the Moon and the Sun on Earth.




▲ **Figure 4.21**

2. During the third quarter phase of the Moon, Earth, the Moon, and the Sun are positioned as shown in Figure 4.22. Calculate the net gravitational force exerted by both the Moon and the Sun on Earth.




▲ **Figure 4.22**

### Answers

1. $3.54 \times 10^{22}$ N [toward Sun's centre]
2. $3.52 \times 10^{22}$ N [0.3°]

Calculate $F_g$ exerted by the Sun on Earth using Newton's law of gravitation.

$$(F_g)_2 = \frac{Gm_{Earth}m_{Sun}}{(r_{E\ to\ S})^2}$$

$$= \frac{\left(6.67 \times 10^{-11}\ \frac{N \cdot m^2}{\cancel{kg^2}}\right)(5.97 \times 10^{24}\ \cancel{kg})(1.99 \times 10^{30}\ \cancel{kg})}{(1.50 \times 10^{11}\ m)^2}$$

$$= 3.522 \times 10^{22}\ N$$

$$\vec{F}_{g_2} = 3.522 \times 10^{22}\ N\ [\text{toward Sun's centre}]$$

Find the net gravitational force on Earth.

$$\vec{F}_{g_{net}} = \vec{F}_{g_1} + \vec{F}_{g_2}$$

$$F_{g_{net}} = F_{g_1} + F_{g_2}$$

$$= -1.985 \times 10^{20}\ N + 3.522 \times 10^{22}\ N$$

$$= 3.50 \times 10^{22}\ N$$

$$\vec{F}_{g_{net}} = 3.50 \times 10^{22}\ N\ [\text{toward Sun's centre}]$$

***Paraphrase***

The net gravitational force on Earth due to the Sun and Moon during a lunar eclipse is $3.50 \times 10^{22}$ N [toward Sun's centre].

## The Role of Gravitational Force on Earth's Tides

Newton used the concept of gravitational force to account for Earth's tides. Although he correctly identified the gravitational force exerted by the Moon and the Sun on Earth as the major cause, a complete understanding of tides must take into account other factors as well.

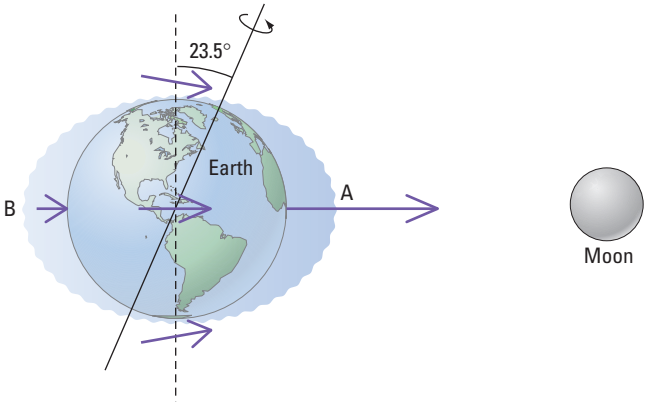
The height of the tides varies depending on the location on Earth (Figure 4.23). In the middle of the Pacific Ocean, the difference between high and low tides is about 0.5 m. But along the coastline of the continents, the difference may be considerably greater.

▲ **Figure 4.23** The tides in the Bay of Fundy are the highest in the world. In some locations, the water level rises up to 18 m between low and high tides.

Some factors that affect tides are

- the shape of the coastline
- the topography of the ocean floor near the coastline
- friction between Earth and the ocean water
- Earth's position in its orbit around the Sun
- Earth's rotation about its axis
- the tilt of Earth's axis
- the alignment of Earth, the Moon, and the Sun

First consider only the Moon's influence on Earth. Since the Moon exerts a gravitational force on Earth, the Moon is in a sense pulling Earth closer to it. So the land mass and ocean water on Earth are all "falling" toward the Moon. In Figure 4.24, this gravitational force is greatest at side A, then decreases at the midpoints of A and B, and is least at side B, because the magnitude of the gravitational force varies inversely with the square of the separation distance.




▲ **Figure 4.24** Earth experiences high tides on sides A and B at the same time. The vectors show the relative gravitational force exerted by the Moon on a test mass at various locations near Earth's surface.

The bulges at A and B are the **high tides**. The **low tides** occur at the midpoints of A and B. The bulge at B occurs because the land mass of Earth at B is pulled toward the Moon, leaving the ocean water behind.

Next consider the fact that Earth rotates on its axis once every 24 h. As the bulges remain fixed relative to the Moon, Earth rotates underneath those bulges. So at a given location on Earth's surface, a high tide is replaced by a low tide about 6 h later, followed again by a high tide about 6 h later, and so on. These time intervals are actually a bit longer than 6 h because the Moon is orbiting Earth every $27\frac{1}{3}$ days with respect to distant stars, and the Moon is taking the bulges along with it.

Now consider that Earth is tilted on its axis. When the northern hemisphere is under the bulge at A, the southern hemisphere is under the bulge at B. So the high tides that are 12 h apart are not equally high, and low tides that are 12 h apart are not equally low.

Example 4.4 demonstrates how to calculate the gravitational force exerted by the Moon on 1.0000 kg of water at A.

**e WEB**

Newspapers in cities near an ocean, such as Halifax, often publish tidal charts listing the times of local high and low tides. Find an example of a tidal chart and suggest how different people would find this information useful. Begin your search at www.pearsoned.ca/school/physicssource.

**info BIT**

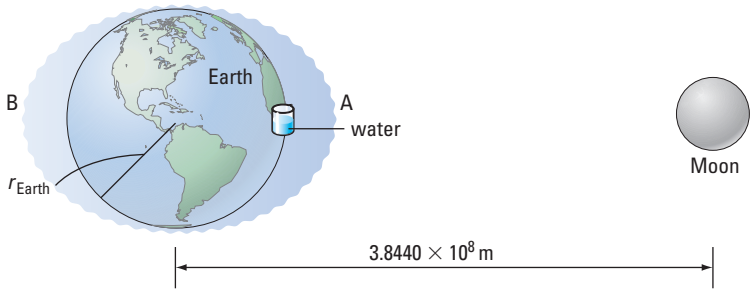
Io, one of Jupiter's moons, has tidal bulges of up to 100 m compared to typical tidal bulges of 1 m on Earth.

## Example 4.4

Calculate the gravitational force exerted by the Moon on 1.0000 kg of water at A (Figure 4.25). Use $G = 6.672\ 59 \times 10^{-11}\ \text{N·m}^2/\text{kg}^2$.

| Celestial Body | Mass* (kg) | Equatorial Radius* (m) | Mean Separation Distance from Earth* (m) |
|---|---|---|---|
| Earth | $5.9742 \times 10^{24}$ | $6.3781 \times 10^{6}$ | — |
| Earth's Moon | $7.3483 \times 10^{22}$ | $1.7374 \times 10^{6}$ | $3.8440 \times 10^{8}$ |

*Source: Jet Propulsion Laboratory, California Institute of Technology (See JPL link at www.pearsoned.ca/school/physicssource.)




▲ **Figure 4.25**

### *Given*

$m_w = 1.0000$ kg

$m_{Earth} = 5.9742 \times 10^{24}$ kg $\quad r_{Earth} = 6.3781 \times 10^{6}$ m

$m_{Moon} = 7.3483 \times 10^{22}$ kg $\quad r_{Moon} = 1.7374 \times 10^{6}$ m

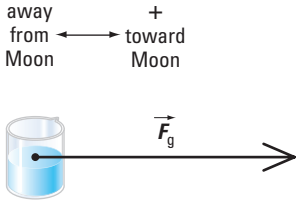
$r_{E\ to\ M} = 3.8440 \times 10^{8}$ m

### *Required*

gravitational force exerted by Moon on water ($\vec{F}_g$)

### *Analysis and Solution*

Draw a free-body diagram for the water showing only $\vec{F}_g$ due to the Moon (Figure 4.26).




▲ **Figure 4.26**

## Practice Problem

1. Using the value of $G$ given in Example 4.4, calculate the gravitational force exerted by the Moon on 1.0000 kg of water
   (a) at the midpoints of A and B, and
   (b) at B.

### Answer

1. (a) $3.3183 \times 10^{-5}$ N [toward Moon's centre]
   (b) $3.2109 \times 10^{-5}$ N [toward Moon's centre]

Find the separation distance between the water and the Moon.

$$r = r_{\text{E to M}} - r_{\text{Earth}}$$
$$= 3.8440 \times 10^{8}\ \text{m} - 6.3781 \times 10^{6}\ \text{m}$$
$$= 3.780\ 22 \times 10^{8}\ \text{m}$$

Calculate $F_g$ exerted by the Moon on the water using Newton's law of gravitation.

$$F_g = \frac{G m_w m_{\text{Moon}}}{r^2}$$
$$= \frac{\left(6.672\ 59 \times 10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\cancel{\text{kg}^2}}\right)(1.0000\ \cancel{\text{kg}})(7.3483 \times 10^{22}\ \cancel{\text{kg}})}{(3.780\ 22 \times 10^{8}\ \text{m})^2}$$
$$= 3.4312 \times 10^{-5}\ \text{N}$$
$$\vec{F}_g = 3.4312 \times 10^{-5}\ \text{N [toward Moon's centre]}$$

***Paraphrase***

The gravitational force exerted by the Moon on the water at A is $3.4312 \times 10^{-5}$ N [toward Moon's centre].

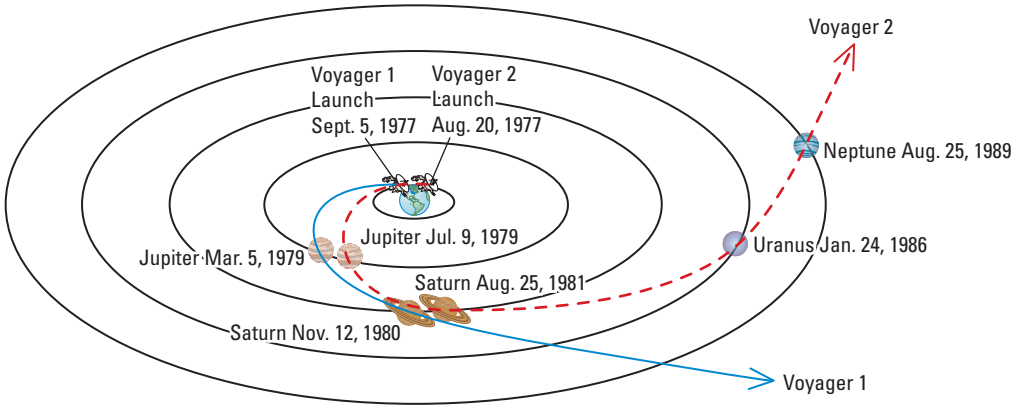
## The Role of Gravitational Force on Interplanetary Travel

Scientists who plan space missions take advantage of the gravitational force exerted by planets and other celestial bodies to change the speed and direction of spacecraft. Distances between celestial bodies are huge compared to distances on Earth. So a space probe leaving Earth to study Jupiter and Saturn and their moons would take many years to arrive there.

Scientists have to calculate the position and velocity of all the celestial bodies that will affect the motion of the probe many years in advance. If several planets are moving in the same direction and their positions are aligned, a space probe launched from Earth can arrive at its destination many years sooner, provided the probe moves near as many of those planets as possible (Figure 4.27).

***info* BIT**

The Voyager mission was intended to take advantage of a geometric alignment of Jupiter, Saturn, Uranus, and Neptune. This arrangement occurs approximately every 175 years. By using the concept of gravity assist, the flight time to Neptune was reduced from 30 to 12 years, and a minimum of onboard propellant on the spacecraft was required.



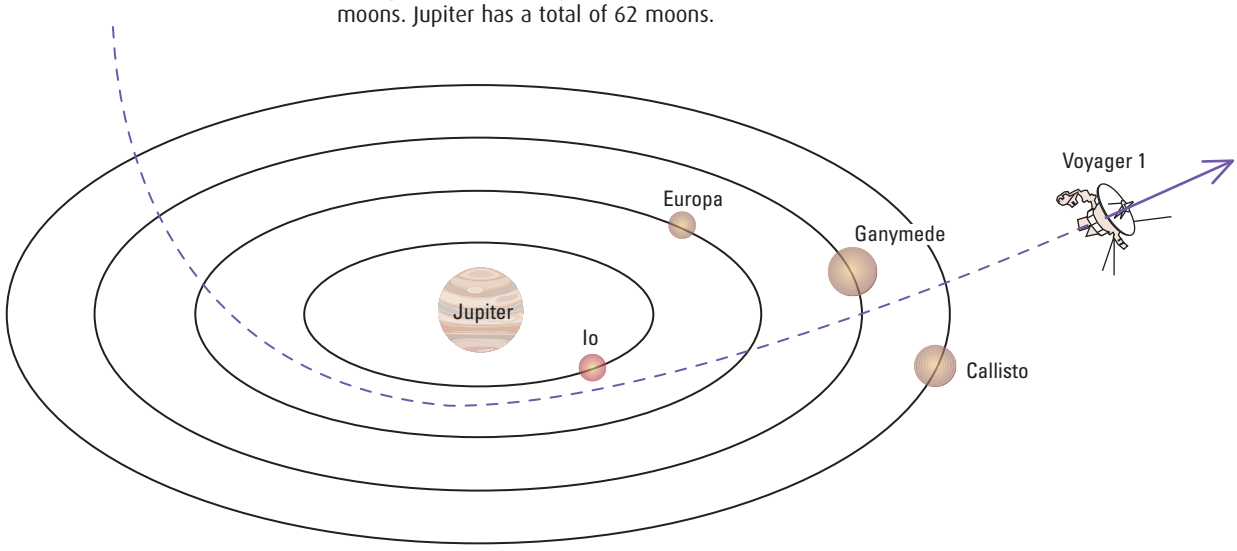

▲ **Figure 4.27** Both Voyager spacecraft were launched from Cape Canaveral, Florida, in 1977. Voyager 1 had close encounters with Jupiter and Saturn, while Voyager 2 flew by all four of the gaseous planets in the solar system.

**info BIT**

Voyager 1 identified nine active volcanoes on Io, one of Jupiter's moons. Up until that point, scientists knew of no other celestial body in the solar system, other than Earth, that has active volcanoes. According to Voyager 1's instruments, the debris being ejected from Io's volcanoes had a speed of $1.05 \times 10^3$ m/s compared to speeds of 50 m/s at Mount Etna on Earth.

Each time the probe gets near enough to one of these planets, the gravitational field of the planet causes the path of the probe to curve (Figure 4.28). The planet deflects the space probe and, if the planet is moving in the same direction as the probe, the speed of the probe after its planetary encounter will increase. The use of the gravitational force exerted by celestial bodies to reduce interplanetary travel times is called **gravity assist**.

▼ **Figure 4.28** Voyager 1 passed close to Io, Ganymede, and Callisto, three of Jupiter's moons. Jupiter has a total of 62 moons.




## THEN, NOW, AND FUTURE — Small Steps Lead to New Models

When Newton began his study of gravity in 1665, he was not the first person to tackle the challenge of explaining planetary motion.

Ancient Greek astronomer Ptolemy (Claudius Ptolemaeus, 2nd century A.D.) and eventually Nicolaus Copernicus (1473–1543) proposed two different models of the solar system: one Earth-centred and the other Sun-centred. Later Johannes Kepler (1571–1630) developed three empirical laws describing planetary motion using astronomical data compiled by astronomer Tycho Brahe (1546–1601).

Kepler's laws confirmed that a Sun-centred system is the correct model, because it was possible to predict the correct position of planets. However, Kepler was unable to explain *why* planets move.

Newton was the first scientist to explain the motion of planets in terms of forces. By using his three laws of motion and his law of gravitation, Newton derived Kepler's laws, providing further evidence of the validity of his force laws and of the model of a Sun-centred solar system.

Newton was able to develop his law of gravitation because many scientists before him developed theories and made observations about planetary motion. Newton's laws and the concept of gravity could describe the motion of objects on Earth and throughout the universe.

This was a tremendous breakthrough because up until that time, scientists were unable to predict or explain motion. While Newton's model can still be used today for most everyday situations, scientists have further modified it. The process of developing new models and theories has helped scientists tackle questions about the universe in ways Newton could never have imagined.

### Questions

1. Research the scientific developments that led to Newton's law of gravitation.
2. What are some benefits of developing new scientific models and theories?

## 4.2 Check and Reflect

### Knowledge

**1.** Why is $G$ called a "universal" constant?

**2.** Describe how a torsion balance can be used to measure the constant $G$.

**3.** Suppose $F_g$ is the magnitude of the gravitational force between two people with a separation distance of 1.0 m. How would $F_g$ change if

(a) the separation distance became 2.0 m?

(b) one person was joined by an equally massive friend while at this 2.0-m separation distance?

### Applications

**4.** The Moon has a mass of $7.35 \times 10^{22}$ kg and its equatorial radius is $1.74 \times 10^6$ m. Earth's mass is $5.97 \times 10^{24}$ kg and its equatorial radius is $6.38 \times 10^6$ m.

(a) Calculate the magnitude of the gravitational force exerted by

(i) the Moon on a 100-kg astronaut standing on the Moon's surface, and

(ii) Earth on a 100-kg astronaut standing on Earth's surface.

(b) Explain why the values of $F_g$ in part (a) are different.

**5.** Mars has two moons, Deimos and Phobos, each named after an attendant of the Roman war god Mars. Deimos has a mass of $1.8 \times 10^{15}$ kg and its mean distance from Mars is $2.3 \times 10^7$ m. Phobos has a mass of $1.1 \times 10^{16}$ kg and its mean distance from Mars is $9.4 \times 10^6$ m.

(a) Without doing any calculations, predict which moon will exert a greater gravitational force on Mars. Explain your reasoning.

(b) Check your prediction in part (a) by calculating the magnitude of the gravitational force exerted by each moon on Mars. Mars' mass is $6.42 \times 10^{23}$ kg. Show complete solutions.

**6.** Suppose the equatorial radius of Earth was the same as the Moon, but Earth's mass remained the same. The Moon has an equatorial radius of $1.74 \times 10^6$ m. Earth's mass is $5.97 \times 10^{24}$ kg and its equatorial radius is $6.38 \times 10^6$ m.

(a) Calculate the gravitational force that this hypothetical Earth would exert on a 1.00-kg object at its surface.

(b) How does the answer in part (a) compare to the actual gravitational force exerted by Earth on this object?

### Extensions

**7.** Prepare a problem involving Newton's law of gravitation for each situation. Work with a partner to solve each problem, and discuss the steps you use.

(a) Choose the values of the two masses and the separation distance.

(b) Use values of $m_A$, $m_B$, and $r$ that are multiples of those in part (a). Use proportionalities to solve the problem.

**8.** During Newton's time, scientists often

- worked alone and contact with other scientists working on similar problems was difficult.
- had very strong religious beliefs and some scientists, such as Kepler, were members of the clergy.
- were knowledgeable in many different fields. Newton, for example, spent many years doing chemistry.

(a) In paragraph form, assess the impacts that these factors might have had on science in Newton's day.

(b) In a paragraph, describe the effects these factors might have on scientists today.

**eTEST**

To check your understanding of Newton's law of gravitation, follow the eTest links at www.pearsoned.ca/school/physicssource.

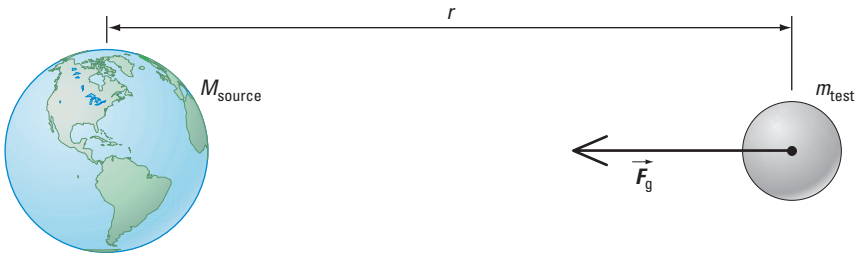
# 4.3 Relating Gravitational Field Strength to Gravitational Force

**info BIT**

A *g* force is a force that causes an acceleration with a magnitude of some multiple of *g*. A force of 2*g* means the magnitude of the acceleration is $2 \times 9.81\ m/s^2 = 19.6\ m/s^2$.

The acceleration due to gravity $\vec{g}$ near or on Earth's surface is about 9.81 m/s² [down]. But where does the value of 9.81 m/s² come from?

Consider the forces acting on a test mass $m_{test}$ some distance above Earth's surface, where Earth has a mass of $M_{source}$. The only force acting on $m_{test}$ is the gravitational force exerted by Earth on the test mass (Figure 4.29).




▲ **Figure 4.29** The gravitational force exerted by Earth on test mass $m_{test}$

**info BIT**

During a roller coaster ride, riders may experience a 4*g* change in acceleration between the top and bottom of a loop. This dramatic change in acceleration causes the thrill and occasional dizziness experienced by riders.

The magnitude of $\vec{F}_g$ can be evaluated two ways: using the concept of weight or using Newton's law of gravitation.

| Weight | Newton's law of gravitation |
|---|---|
| $F_g = m_{test} g$ | $F_g = \dfrac{G m_{test} M_{source}}{r^2}$ |

Since the value of $F_g$ is the same no matter which equation you use, set both equations equal to each other.

$$m_{test}\, g = \frac{G m_{test}\, M_{source}}{r^2}$$

$$g = \frac{G M_{source}}{r^2}$$

So no matter where the test mass is located in the universe, you can calculate the magnitude of the gravitational field strength (or gravitational acceleration) at any distance from a celestial body if you know the mass of the celestial body $M_{source}$ and the separation distance between the centre of the test mass and the celestial body *r*.

Find out the relationship between gravitational field strength and the acceleration due to gravity by doing 4-3 Design a Lab.

## 4-3 Design a Lab

# Comparing Gravitational Field Strength to Gravitational Acceleration

### The Question

What is the relationship between gravitational field strength and the local value of the gravitational acceleration?

### Design and Conduct Your Investigation

State a hypothesis. Then design an experiment. Identify the controlled, manipulated, and responding variables. Review the procedure in 4-2 QuickLab on page 198. List the materials you will use, as well as a detailed procedure. Check the procedure with your teacher and then do the investigation. Analyze your data and form a conclusion. How well did your results agree with your hypothesis?

## How Is Gravitational Field Strength Related to Gravitational Acceleration?

To determine how gravitational field strength is related to gravitational acceleration, use the definition of a newton, $1\ \text{N} = 1\ \text{kg}\cdot\text{m/s}^2$. Then substitute kilogram-metres per second squared for newtons in the equation for gravitational field strength:

$$1\ \frac{\text{N}}{\text{kg}} = 1\ \frac{\cancel{\text{kg}}\cdot\frac{\text{m}}{\text{s}^2}}{\cancel{\text{kg}}}$$

$$= 1\ \frac{\text{m}}{\text{s}^2}$$

Metres per second squared are the units of acceleration. So in terms of units, gravitational field strength and gravitational acceleration are equivalent (Figure 4.30).

◀ **Figure 4.30** Jennifer Heil of Spruce Grove, Alberta, won the gold medal in the women's freestyle skiing moguls in the 2006 Winter Olympics in Turin, Italy. The gravitational field strength at the surface of the Moon is about $\frac{1}{6}$ that at Earth's surface. How would Jennifer's jump on Earth compare with one on the Moon?

## Calculating the Gravitational Acceleration of an Object on Two Celestial Bodies

Example 4.5 demonstrates how to calculate the gravitational acceleration at the equator on Earth's surface and that on the surface of the Moon. These two values are then compared to find the ratio of $g_{Earth}$ to $g_{Moon}$.

To solve the problem requires using data in Table 4.1, which shows the mass and equatorial radius of the Sun, the Moon, and each planet in the solar system.

**▼ Table 4.1** Masses and Radii for Celestial Bodies in the Solar System*

| Celestial Body | Mass (kg) | Equatorial Radius (m) |
|---|---|---|
| Sun | $1.99 \times 10^{30}$ | $6.96 \times 10^{8}$ |
| Mercury | $3.30 \times 10^{23}$ | $2.44 \times 10^{6}$ |
| Venus | $4.87 \times 10^{24}$ | $6.05 \times 10^{6}$ |
| Earth | $5.97 \times 10^{24}$ | $6.38 \times 10^{6}$ |
| Earth's Moon | $7.35 \times 10^{22}$ | $1.74 \times 10^{6}$ |
| Mars | $6.42 \times 10^{23}$ | $3.40 \times 10^{6}$ |
| Jupiter | $1.90 \times 10^{27}$ | $7.15 \times 10^{7}$ |
| Saturn | $5.68 \times 10^{26}$ | $6.03 \times 10^{7}$ |
| Uranus | $8.68 \times 10^{25}$ | $2.56 \times 10^{7}$ |
| Neptune | $1.02 \times 10^{26}$ | $2.48 \times 10^{7}$ |

*Source: Jet Propulsion Laboratory, California Institute of Technology (See JPL link at www.pearsoned.ca/school/physicssource.)

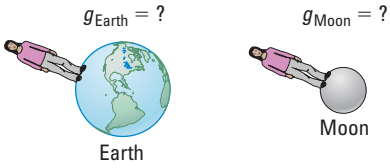
**eWEB**

Globular clusters are groups of about 1 000 000 stars that are bound together by gravity. Find out who discovered the first cluster and how many have been identified so far. Research the approximate size and shape of a globular cluster and the forces involved in its formation. Summarize your findings. Begin your search at www.pearsoned.ca/school/physicssource.

## Example 4.5

(a) Calculate the magnitude of the gravitational acceleration of an object at the equator on the surface of Earth and the Moon (Figure 4.31). Refer to Table 4.1 above.

(b) Determine the ratio of $g_{Earth}$ to $g_{Moon}$. How different would your weight be on the Moon?




**▲ Figure 4.31**

***Given***

$m_{Earth} = 5.97 \times 10^{24}$ kg $\quad r_{Earth} = 6.38 \times 10^{6}$ m

$m_{Moon} = 7.35 \times 10^{22}$ kg $\quad r_{Moon} = 1.74 \times 10^{6}$ m

***Required***

(a) magnitude of gravitational acceleration at equator on Earth and the Moon ($g_{Earth}$ and $g_{Moon}$)

(b) ratio of $g_{Earth}$ to $g_{Moon}$

### *Analysis and Solution*

(a) Use the equation $g = \dfrac{GM_{source}}{r^2}$ to calculate the magnitude of the gravitational field strength on each celestial body. The magnitude of the gravitational acceleration is numerically equal to the magnitude of the gravitational field strength.

Earth

$$g_{Earth} = \frac{Gm_{Earth}}{(r_{Earth})^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \dfrac{N \cdot \cancel{m^2}}{\cancel{kg^2}}\right)(5.97 \times 10^{24}\ \cancel{kg})}{(6.38 \times 10^{6}\ \cancel{m})^2}$$

$$= 9.783\ N/kg$$

$$= 9.783\ m/s^2$$

The Moon

$$g_{Moon} = \frac{Gm_{Moon}}{(r_{Moon})^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \dfrac{N \cdot \cancel{m^2}}{\cancel{kg^2}}\right)(7.35 \times 10^{22}\ \cancel{kg})}{(1.74 \times 10^{6}\ \cancel{m})^2}$$

$$= 1.619\ N/kg$$

$$= 1.619\ m/s^2$$

(b) Calculate the ratio of $g_{Earth}$ to $g_{Moon}$.

$$\frac{g_{Earth}}{g_{Moon}} = \frac{9.783\ \dfrac{\cancel{m}}{\cancel{s^2}}}{1.619\ \dfrac{\cancel{m}}{\cancel{s^2}}}$$

$$= 6.04$$

### *Paraphrase*

(a) The magnitude of the gravitational acceleration at the equator on the surface of Earth is 9.78 $m/s^2$ and of the Moon is 1.62 $m/s^2$.

(b) The ratio of $g_{Earth}$ to $g_{Moon}$ is 6.04. So your weight would be about 6 times less on the surface of the Moon than on Earth.

## Practice Problems

1. A satellite orbits Earth at a distance of $3r_{Earth}$ above Earth's surface. Refer to Table 4.1 on page 218.
   (a) How many Earth radii is the satellite from Earth's centre?
   (b) What is the magnitude of the gravitational acceleration of the satellite?
2. An 80.0-kg astronaut is in orbit $3.20 \times 10^4$ km from Earth's centre. Refer to Table 4.1 on page 218.
   (a) Calculate the magnitude of the gravitational field strength at the location of the astronaut.
   (b) What would be the magnitude of the gravitational field strength if the astronaut is orbiting the Moon with the same separation distance?
3. The highest satellites orbit Earth at a distance of about $6.6r_{Earth}$ from Earth's centre. What would be the gravitational force on a 70-kg astronaut at this location?

### Answers

1. (a) $4r_{Earth}$
   (b) $6.14 \times 10^{-1}$ $m/s^2$
2. (a) $3.89 \times 10^{-1}$ N/kg
   (b) $4.79 \times 10^{-3}$ N/kg
3. 16 N [toward Earth's centre]

## Calculating the Weight of an Object on Mars

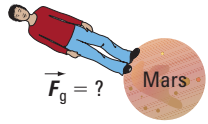
The equation $g = \dfrac{GM_{source}}{r^2}$ can be used to calculate the weight of an object on any celestial body. In Example 4.6, the weight of a student on Mars is calculated. This quantity is then compared with the student's weight on Earth's surface.

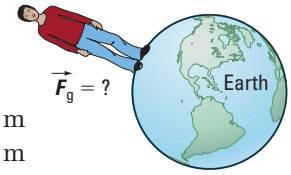
An interesting application of the variation in the weight of an object involves the Mars rover (Figure 4.32). The rover had a mass of about 175 kg, but on the surface of Mars, the rover weighed about 2.5 times less than on Earth's surface. The rover was designed to avoid inclines greater than 30° but because $g_{Mars} < g_{Earth}$, the rover could travel farther up an incline on Mars than on Earth using the same battery charge.

▲ **Figure 4.32** In full sunlight, a 140-W battery enabled the Mars rover to travel about 100 m per day on level ground with an average speed of 1.0 cm/s between charges.

## Example 4.6

(a) What is the mass of a 60.0-kg student on Mars and on Earth?
(b) What is the student's weight at the equator on the surface of Mars and of Earth (Figure 4.33). Refer to Table 4.1 on page 218.




▲ **Figure 4.33**

### *Given*

(a) $m_s = 60.0 \text{ kg}$
(b) $m_{Mars} = 6.42 \times 10^{23} \text{ kg}$ $r_{Mars} = 3.40 \times 10^6 \text{ m}$
$m_{Earth} = 5.97 \times 10^{24} \text{ kg}$ $r_{Earth} = 6.38 \times 10^6 \text{ m}$

### *Required*

(a) mass on Mars and on Earth ($m$)
(b) weight on Mars and on Earth ($\vec{F}_{g_{Mars}}$ and $\vec{F}_{g_{Earth}}$)

### *Analysis and Solution*

(a) Mass is a scalar quantity and does not depend on location. So the student's mass will be the same on Mars as on Earth.

(b) Use the equation $g = \dfrac{GM_{source}}{r^2}$ to calculate the magnitude of the gravitational field strength on Mars and on Earth.

Mars

$$g_{Mars} = \frac{Gm_{Mars}}{(r_{Mars})^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \dfrac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(6.42 \times 10^{23} \cancel{\text{kg}})}{(3.40 \times 10^6 \cancel{\text{m}})^2}$$

$$= 3.704 \text{ N/kg}$$

Earth

$$g_{Earth} = \frac{Gm_{Earth}}{(r_{Earth})^2}$$

$$= \frac{\left(6.67 \times 10^{-11} \dfrac{\text{N} \cdot \cancel{\text{m}^2}}{\text{kg}^{\cancel{2}}}\right)(5.97 \times 10^{24} \cancel{\text{kg}})}{(6.38 \times 10^6 \cancel{\text{m}})^2}$$

$$= 9.783 \text{ N/kg}$$

Since the direction of $\vec{F}_g$ will be toward the centre of each celestial body, use the scalar equation $F_g = mg$ to find the magnitude of the weight.

Mars

$$F_{g_{Mars}} = m_s g_{Mars}$$
$$= (60.0 \cancel{\text{kg}})\left(3.704 \frac{\text{N}}{\cancel{\text{kg}}}\right)$$
$$= 222 \text{ N}$$

Earth

$$F_{g_{Earth}} = m_s g_{Earth}$$
$$= (60.0 \cancel{\text{kg}})\left(9.783 \frac{\text{N}}{\cancel{\text{kg}}}\right)$$
$$= 587 \text{ N}$$

## Practice Problems

1. What would be the weight of a 22.0-kg dog at the equator on Saturn's surface? Refer to Table 4.1 on page 218.
2. (a) Do you think your skeleton could support your weight on Jupiter?
   (b) Compared to Earth, how much stronger would your bones need to be?
3. (a) What is the magnitude of the gravitational field strength at the equator on Uranus' surface?
   (b) Compared to Earth, how would your weight change on Uranus?

### Answers

1. 246 N [toward Saturn's centre]
2. (a) no
   (b) 2.64 times
3. (a) 8.83 N/kg, (b) 0.903 $F_{g_{Earth}}$

***Paraphrase and Verify***

(a) The mass of the student would be 60.0 kg on both Mars and Earth.

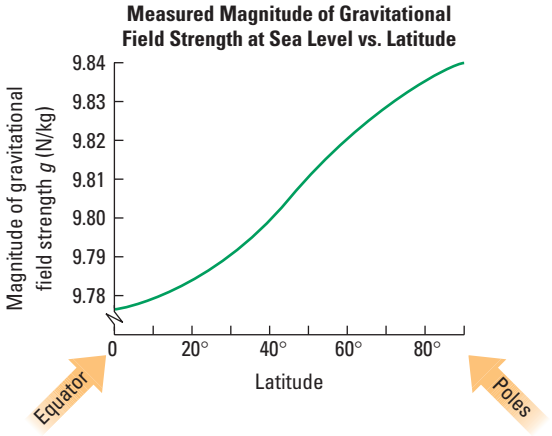
(b) The weight of the student on Mars would be 222 N [toward Mars' centre] and on Earth 587 N [toward Earth's centre]. So the student would weigh about 2.6 times more on Earth than Mars.

## Different Values of Gravitational Field Strength on Earth

For a long time, people thought that the magnitude of the gravitational field strength was constant at any location on Earth's surface. However, scientists discovered that the value of $g$ depends on both latitude and altitude. **Latitude** is the angular distance north or south of the equator. **Altitude** is the elevation of the ground above sea level. Figure 4.34 shows how the magnitude of the gravitational field strength at sea level varies with latitude.

*info* **BIT**

The value 9.81 N/kg is an average of the magnitude of the gravitational field strength at different locations on Earth's surface.



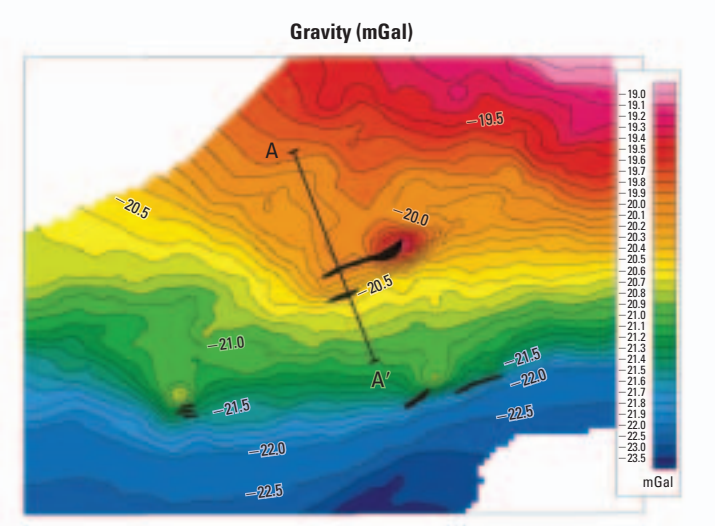

◀ **Figure 4.34** Gravitational field strength at sea level as a function of latitude. At what location on Earth's surface would you weigh the least? The most?

The value of $g$ increases as you move toward either the North or South Pole, because Earth is not a perfect sphere. It is flatter at the poles and it bulges out slightly at the equator. In fact, Earth's radius is 21 km greater at the equator than at the poles. So an object at the equator is farther away from Earth's centre than if the object were at the North Pole. Since $g \propto \frac{1}{r^2}$, the farther an object is from Earth's centre, the smaller the value of $g$ will be.

Other factors affect the value of $g$ at Earth's surface. The materials that make up Earth's crust are not uniformly distributed. Some materials, such as gold, are more dense than materials such as zinc. Earth's rotation about its axis also reduces the measured value of $g$, but the closer an object is to the North or South Pole, the less effect Earth's rotation has on $g$.

### *Concept Check*

Leo weighs 638 N [down] in Calgary, Alberta. What are some problems with Leo saying he weighs 638 N [down] anywhere on Earth? What property of matter would he be more accurate to state?




▲ **Figure 4.35** A map showing the location of sulphide deposits in northern New Brunswick (shown in black)

## Applications of the Variation in *g* in Geology

The variation in the value of $g$ on Earth is used to detect the presence of minerals and oil. Geophysicists and geologists use sensitive instruments, called **gravimeters**, to detect small variations in $g$ when they search for new deposits of ore or oil. Gold and silver deposits increase the value of $g$, while deposits of oil and natural gas decrease $g$. Figure 4.35 is an example of a map that shows different measured values of $g$ as lines, where each line represents a specific value of $g$.

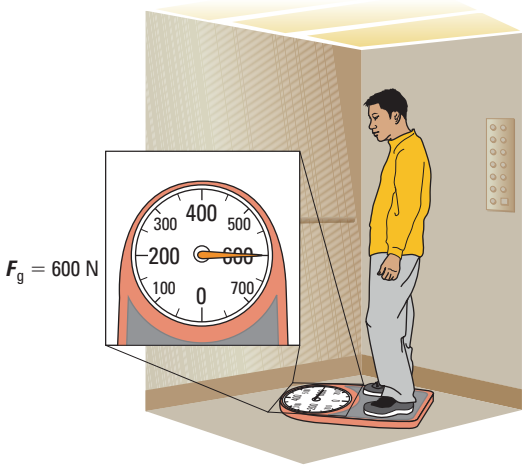
**true weight:** gravitational force acting on an object that has mass

## True Weight vs. Apparent Weight

So far, you used the equation $\vec{F}_g = m\vec{g}$ to calculate the weight of an object at any location in the universe. The gravitational force that you calculate with this equation is really called the **true weight**, $\vec{F}_g$, of an object.

Suppose a student is standing on a scale calibrated in newtons in an elevator (Figure 4.36). If the elevator is at rest or is moving at constant velocity, the scale reads 600 N.

▲ **Figure 4.37** The free-body diagram for the student in Figure 4.36



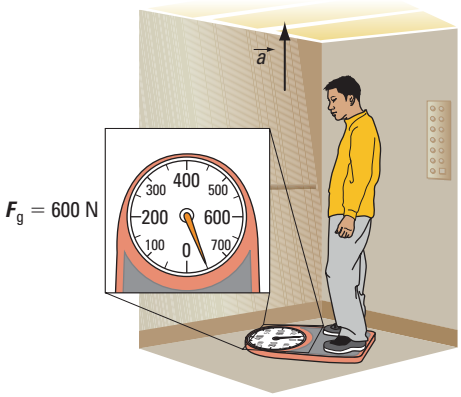

▲ **Figure 4.36** The elevator and student are either at rest or moving at constant velocity.

Using the free-body diagram for the student (Figure 4.37), the equation for the net force on the student is

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_N$$
$$0 = \vec{F}_g + \vec{F}_N$$
$$0 = F_g + F_N$$
$$0 = -mg + F_N$$
$$F_N = mg$$

So when $\vec{F}_{net} = 0$ N on the student, the magnitude of the normal force is equal to the magnitude of the student's weight.

Now suppose the elevator is accelerating up uniformly (Figure 4.38). In this situation, the scale reads 750 N.



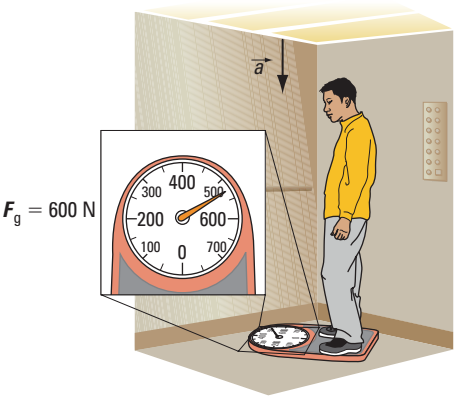

▲ **Figure 4.38** The elevator and student are accelerating up uniformly.

To understand why the reading on the scale is different, draw the free-body diagram for the student (Figure 4.39) and write the equation for the net force:

$$\vec{F}_{net} = \vec{F}_g + \vec{F}_N$$
$$m\vec{a} = \vec{F}_g + \vec{F}_N$$
$$\vec{F}_N = m\vec{a} - \vec{F}_g$$
$$= m\vec{a} - m\vec{g}$$
$$= m(\vec{a} - \vec{g})$$

The equation for $\vec{F}_N$ is valid whether the student is accelerating up or down. In Figure 4.38, the student feels heavier than usual because the scale is pushing up on him with a force greater than $mg$.

If the elevator is accelerating down uniformly, the scale reads 525 N (Figure 4.40). As before, $\vec{F}_N = m(\vec{a} - \vec{g})$ but this time $\vec{a}$ and $\vec{g}$ are in the same direction. So $F_N$ is less than $mg$, and the student feels lighter than usual.




▲ **Figure 4.40** The elevator and student are accelerating down uniformly.

▲ **Figure 4.39** The free-body diagram for the student in Figure 4.38

**eSIM**

Calculate the true weight, normal force, and apparent weight of a person during an elevator ride. Follow the eSim links at www.pearsoned.ca/school/physicssource.

**apparent weight:** negative of the normal force acting on an object

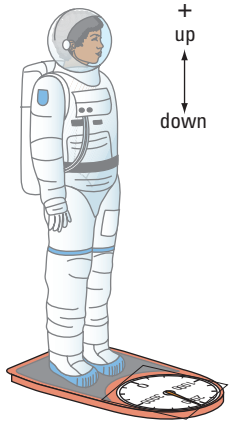
The quantity $-\vec{F}_N$ is called the **apparent weight**, $\vec{w}$, of an object. For the situations shown in Figures 4.38 and 4.40 on page 223, the equation for the apparent weight of the student is

$$\begin{aligned}\vec{w} &= -\vec{F}_N \\ &= -m(\vec{a} - \vec{g}) \\ &= m(\vec{g} - \vec{a})\end{aligned}$$

Example 4.7 demonstrates how to calculate the true weight and apparent weight of an astronaut in a rocket during liftoff on Earth's surface.

## Example 4.7

A 100.0-kg astronaut in a spacesuit is standing on a scale in a rocket (Figure 4.41). The acceleration of the rocket is 19.6 m/s² [up]. Calculate her true weight and apparent weight during liftoff on Earth. The acceleration due to gravity on Earth's surface is 9.81 m/s² [down].




▲ **Figure 4.41**

### *Given*

$m = 100.0$ kg

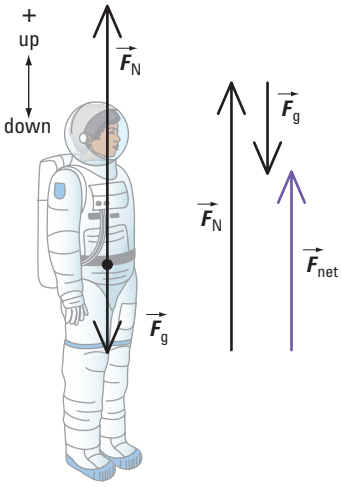
$\vec{a} = 19.6$ m/s² [up] $\quad \vec{g} = 9.81$ m/s² [down]

### *Required*

true weight and apparent weight during liftoff ($\vec{F}_g$ and $\vec{w}$)

### *Analysis and Solution*

Draw a free-body diagram and a vector addition diagram for the astronaut (Figure 4.42).




◀ **Figure 4.42**

Use the equation $\vec{F}_g = m\vec{g}$ to find the astronaut's true weight.

$$\begin{aligned}\vec{F}_g &= m\vec{g} \\ &= (100.0 \text{ kg})\left(-9.81 \frac{\text{m}}{\text{s}^2}\right) \\ &= -9.81 \times 10^2 \text{ N}\end{aligned}$$

The astronaut is not accelerating left or right. So in the horizontal direction, $\vec{F}_{net} = 0$ N.

For the vertical direction, write an equation to find the net force on the astronaut.

$\vec{F}_{net} = \vec{F}_N + \vec{F}_g$

Apply Newton's second law.

$$\begin{aligned} m\vec{a} &= \vec{F}_N + \vec{F}_g \\ \vec{F}_N &= m\vec{a} - \vec{F}_g \\ F_N &= ma - F_g \\ &= (100.0\text{ kg})\left(19.6\ \frac{\text{m}}{\text{s}^2}\right) - (-9.81 \times 10^2\text{ N}) \\ &= (100.0\text{ kg})\left(19.6\ \frac{\text{m}}{\text{s}^2}\right) + 9.81 \times 10^2\text{ N} \\ &= 2.94 \times 10^3\text{ N} \\ \vec{F}_N &= 2.94 \times 10^3\text{ N [up]} \end{aligned}$$

Use the equation $\vec{w} = -\vec{F}_N$ to find the astronaut's apparent weight.

$$\begin{aligned} \vec{w} &= -\vec{F}_N \\ &= -(2.94 \times 10^3\text{ N}) \\ &= -2.94 \times 10^3\text{ N} \end{aligned}$$

***Paraphrase***

During liftoff, the astronaut's true weight is $9.81 \times 10^2$ N [down] and her apparent weight is $2.94 \times 10^3$ N [down].

## Practice Problems

1. In Example 4.7, draw the free-body diagram for the scale during liftoff.
2. Suppose the rocket in Example 4.7 has an acceleration of 19.6 m/s² [down] while it is near Earth's surface. What will be the astronaut's apparent weight and true weight?

### Answers

1. See page 898.
2. $9.79 \times 10^2$ N [down], $9.81 \times 10^2$ N [down]

In Example 4.8, an astronaut is accelerating in deep space, a location in which the gravitational force acting on an object is not measurable. So in deep space, $\vec{F}_g = 0$.

## Example 4.8

Refer to Example 4.7 on pages 224 and 225. What is the magnitude of the astronaut's true weight and apparent weight if the rocket is in deep space? The magnitude of the acceleration of the rocket is 19.6 m/s².

***Given***

$m = 100.0$ kg

magnitude of $\vec{a} = 19.6$ m/s² $\quad\quad \vec{g} = 0$ m/s²

***Required***

magnitude of true weight and apparent weight in deep space ($\vec{F}_g$ and $\vec{w}$)

***Analysis and Solution***

In deep space, the mass of the astronaut is still 100.0 kg, but $\vec{g} = 0$ m/s².

So $\quad \vec{F}_g = m\vec{g}$

$\quad\quad = 0$ N

The astronaut is not accelerating left or right.

So in the horizontal direction, $\vec{F}_{net} = 0$ N.

## Practice Problems

1. An 80.0-kg astronaut is standing on a scale in a rocket leaving the surface of the Moon. The acceleration of the rocket is 12.8 m/s² [up]. On the Moon, $\vec{g} = 1.62$ N/kg [down]. Calculate the magnitude of the true weight and apparent weight of the astronaut
   (a) during liftoff, and
   (b) if the rocket has the same acceleration in deep space.
2. A 60.0-kg astronaut is standing on a scale in a rocket about to land on the surface of Mars. The acceleration of the rocket is 11.1 m/s² [down]. Refer to Table 4.1 on page 218. Calculate the true weight and apparent weight of the astronaut
   (a) as the rocket lands, and
   (b) if the rocket is accelerating at 7.38 m/s² [up] when leaving Mars.

For the vertical direction, write an equation to find the net force on the astronaut. Refer to the free-body diagram in Figure 4.42 on page 224.

$$\begin{aligned}\vec{F}_{net} &= \vec{F}_g + \vec{F}_N \\ &= 0 + \vec{F}_N \\ &= \vec{F}_N\end{aligned}$$

Apply Newton's second law.

$$\vec{F}_N = m\vec{a}$$

$$\begin{aligned}F_N &= ma \\ &= (100.0\text{ kg})\left(19.6\ \frac{\text{m}}{\text{s}^2}\right) \\ &= 1.96 \times 10^3\text{ N}\end{aligned}$$

Use the equation $\vec{w} = -\vec{F}_N$ to find the astronaut's apparent weight.

$$\begin{aligned}\vec{w} &= -\vec{F}_N \\ &= -(1.96 \times 10^3\text{ N}) \\ &= -1.96 \times 10^3\text{ N}\end{aligned}$$

***Paraphrase***

In deep space, the astronaut's true weight is zero and the magnitude of her apparent weight is $1.96 \times 10^3$ N.

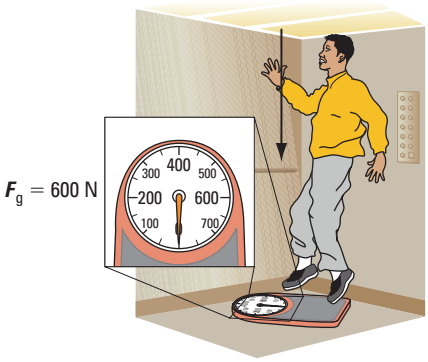
**Answers**

1. (a) ($\vec{F}_g$) 130 N, ($\vec{w}$) $1.15 \times 10^3$ N
   (b) ($\vec{F}_g$) 0 N, ($\vec{w}$) $1.02 \times 10^3$ N
2. (a) ($\vec{F}_g$) 224 N [down], ($\vec{w}$) 442 N [down]
   (b) 666 N [down]

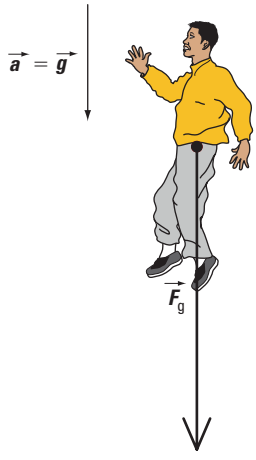
## Free Fall

**free fall:** situation in which the only force acting on an object that has mass is the gravitational force

Let's revisit the elevator scenario on pages 222 and 223. Suppose the elevator cable breaks (Figure 4.43). Assuming that frictional forces are negligible, the elevator, student, and scale all fall toward Earth with an acceleration of $g$. The student is now in **free fall**, the condition in which the only force acting on an object is $\vec{F}_g$.




▲ **Figure 4.43** The elevator, student, and scale are in free fall.




▲ **Figure 4.44** The free-body diagram for the student in Figure 4.43

To understand free fall, draw the free-body diagram for the student (Figure 4.44) and write the equation for the net force:

$$\vec{F}_{net} = \vec{F}_g$$

$$m\vec{a} = \vec{F}_g$$

$$\cancel{m}\vec{a} = \cancel{m}\vec{g}$$

$$\vec{a} = \vec{g}$$

So in free fall, $\vec{a} = \vec{g}$ and both the student and the scale are accelerating at $g$ downward. In Figure 4.43 on page 226, the scale reads zero because it no longer exerts a normal force on the student, so $\vec{F}_N = 0$. Since $\vec{F}_N = 0$, the student's apparent weight is also zero. Sometimes an object in free fall is described as being "weightless." However, this description is incorrect. In free fall, $\vec{F}_N = \vec{w} = 0$ but $\vec{F}_g \neq 0$.

Observe the motion of water in a cup while in free fall by doing 4-4 QuickLab.

## 4-4 QuickLab

# Water in Free Fall

### Problem

What is the motion of water in a cup when the cup is dropped from several metres above Earth's surface?

### Materials

paper cup
pointed pen or pencil
water
dishpan
stepladder

### Procedure

1. Make two holes on opposite sides of the cup near the bottom using the pen or pencil. Cover the holes with your thumb and forefinger. Then fill the cup with water.

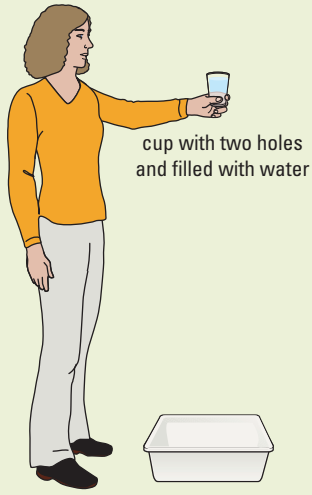
**eLAB**

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

**CAUTION: Do this activity outside. Have someone steady the ladder and be careful when climbing it.**

2. Hold the cup at shoulder height above a dishpan, and uncover the holes. Observe what happens to the water (Figure 4.45). Have a partner sketch the path the water takes.

3. Repeat step 1 but climb the ladder and drop the cup toward the dishpan from a height of several metres. Observe the motion of the water during the fall.




▲ **Figure 4.45**

### Questions

1. Describe the path and motion of the water
   (a) when the cup was held stationary, and
   (b) when the cup was dropped from the ladder. Give a reason for your observations.

## Weightlessness

Videos transmitted from a space shuttle or the space station often show astronauts floating in their cabin (Figure 4.46). Are the astronauts weightless in space? The answer is no. Then why do they appear to be weightless?

Since the shuttle is some distance above Earth, $g$ is less than its value at Earth's surface, because $g \propto \frac{1}{r^2}$. While the shuttle orbits Earth at high speed in an almost circular path, Earth exerts a gravitational force on the shuttle and everything in it. So the shuttle is able to remain in orbit.

If an astronaut were standing on a scale in the shuttle, the scale would read zero, because the shuttle and everything in it are in free fall. The astronaut would feel "weightless" because the gravitational force exerted by Earth pulls the shuttle and the astronaut toward Earth.

Suppose an astronaut is in a rocket in deep space and the acceleration of the rocket is zero. The astronaut would experience no measurable gravitational forces from any celestial bodies, and the astronaut's acceleration would be zero. In this situation, the astronaut would have a true weight of zero and an apparent weight of zero, a condition called **true weightlessness**.

**true weightlessness:** situation in which $\vec{w} = 0$ for an object and $\vec{F}_g = 0$ on the object

▲ **Figure 4.46** At the altitude of the shuttle, the value of $g$ is about 90% of its value at Earth's surface.

## 4.3 Check and Reflect

### Knowledge

**1.** (a) What is the difference between true weight and apparent weight?

(b) Describe a situation in which the true weight of an object is zero but its apparent weight is not zero.

**2.** A person orbiting Earth in a spacecraft has an apparent weight of zero. Explain if the person still experiences a gravitational force.

**3.** List two factors that affect the magnitude of the gravitational field strength at Earth's surface.

### Applications

**4.** Is there a place in the universe where true weightlessness actually exists? Where or why not?

**5.** Calculate the gravitational field strength at the location of a 70-kg astronaut $2.0r_{\text{Earth}}$ from Earth's centre. Refer to Table 4.1 on page 218.

**6.** Graph the equation $g = \dfrac{Gm_{\text{Moon}}}{r^2}$ using technology. Refer to Student References 5: Graphing Data on pp. 872–874. Plot $g$ on the $y$-axis (range of 0−2.0 N/kg) and $r$ on the $x$-axis (range of $1–5r_{\text{Moon}}$). Toggle through to read values of $g$ corresponding to specific values of $r_{\text{Moon}}$ to answer these questions:

(a) Describe the graph of $g$ vs. $r_{\text{Moon}}$. How is it similar to Figure 4.10 on page 201?

(b) What is the value of $g$

(i) on the surface?

(ii) at $\frac{1}{2}r_{\text{Moon}}$ above the surface?

(iii) at $r_{\text{Moon}}$ above the surface?

(c) At what distance from the Moon's surface is $g$ negligible?

**7.** At the top of Mount Robson in British Columbia, a 7.5-kg turkey weighs 73.6 N [down]. Calculate the magnitude of the gravitational field strength at this location.

**8.** An astronaut in a rocket has an apparent weight of $1.35 \times 10^3$ N [down]. If the acceleration of the rocket is 14.7 m/s$^2$ [up] near Earth's surface, what is the astronaut's true weight? The acceleration due to gravity on Earth's surface is about 9.81 m/s$^2$ [down].

**9.** A 50-kg astronaut experiences an acceleration of $5.0g$ [up] during liftoff.

(a) Draw a free-body diagram for the astronaut during liftoff.

(b) What is the astronaut's true weight and apparent weight?

**10.** Calculate the acceleration of the elevator in Figures 4.38 and 4.40 on page 223.

### Extensions

**11.** Draw a flowchart to summarize the steps needed to find the apparent weight of an object. Refer to Student References 4: Using Graphic Organizers on page 869.

**12.** Research how geophysicists and geologists use gravitational field strength to locate minerals, oil, and natural gas in Canada. Prepare a half-page report on your findings. Begin your search at www.pearsoned.ca/school/physicssource.

**13.** Suppose you are wearing a spacesuit. Where could you walk faster, on Earth or on the Moon? Explain your answer.

**14.** Draw a concept map to identify and link the concepts needed to understand gravitational acceleration and gravitational field strength near a celestial body other than Earth. Refer to Student References 4: Using Graphic Organizers on page 869. Create and solve a problem to demonstrate your understanding of these concepts.

**eTEST**

To check your understanding of gravitational field strength, true weight, apparent weight, and free fall, follow the eTest links at www.pearsoned.ca/school/physicssource.

## Key Terms and Concepts

gravitational force
weight
gravitational mass
action-at-a-distance force
field
gravitational field
gravitational field strength
torsion balance
true weight
apparent weight
free fall
true weightlessness

## Key Equations

True weight: $\vec{F}_g = m\vec{g}$

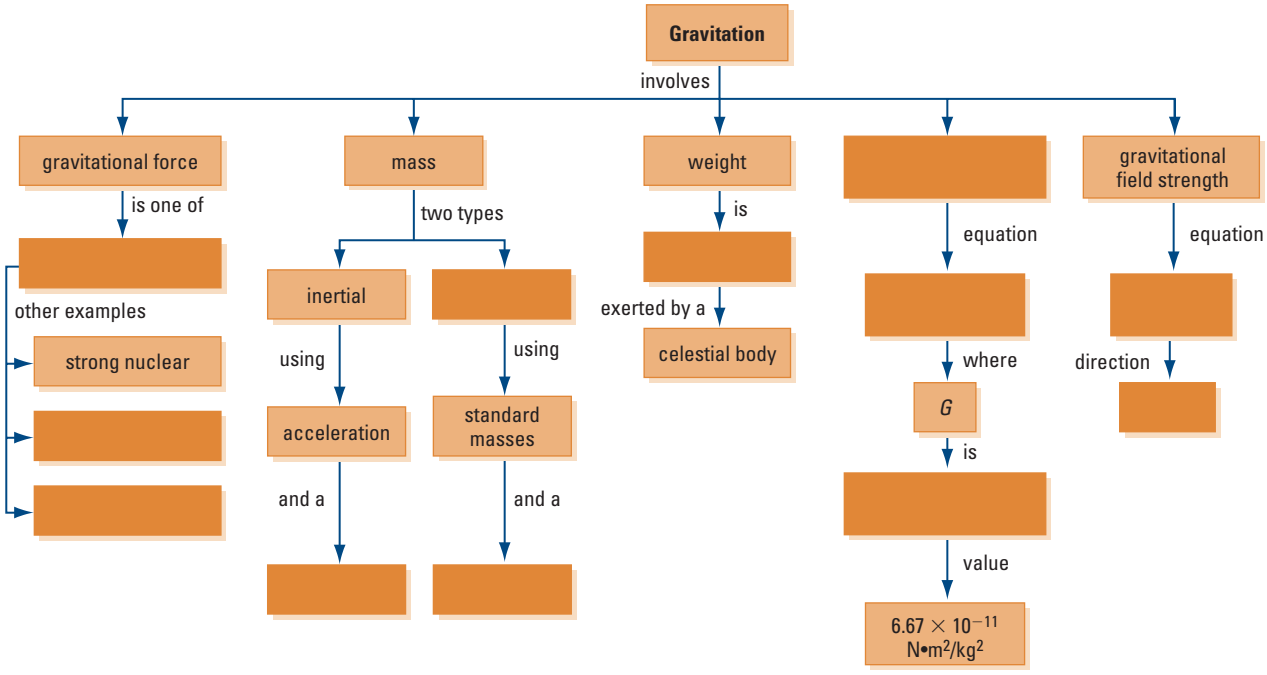
Newton's law of gravitation: $F_g = \dfrac{Gm_A m_B}{r^2}$

Gravitational field strength (or gravitational acceleration): $g = \dfrac{GM_{source}}{r^2}$

Apparent weight: $\vec{w} = -\vec{F}_N$

## Conceptual Overview

The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.




▲ **Figure 4.47**

## Knowledge

**1.** (4.2) What is the significance of the term "universal" in Newton's law of universal gravitation?

**2.** (4.1, 4.3) A brick placed on an equal arm balance requires 5.0 kg to just balance it. When the brick is hung from a spring scale, the scale reads 48 N. The balance, standard masses, spring scale, and brick are moved to a planet where the gravitational field strength is 2.0 times that on Earth. What will be the reading on the balance and on the spring scale in this new location?

**3.** (4.1, 4.3) How does gravitational field strength vary with the mass of a celestial body? Assume that the radius is constant.

**4.** (4.3) The gravitational field strength at Earth's surface is about 9.81 N/kg [down]. What is the gravitational field strength $1.6r_{Earth}$ from Earth's centre?

## Applications

**5.** A 1.0-kg object, initially at rest, is dropped toward Earth's surface. It takes 2.26 s for the object to fall 25 m. Determine how long it takes a 2.0-kg object to fall this distance from rest on Jupiter. Refer to Table 4.1 on page 218.

**6.** Describe the steps you would use to determine the distance from Earth's centre where the gravitational force exerted by Earth on a spacecraft is balanced by the gravitational force exerted by the Moon. Assume that you know the distance from Earth's centre to the centre of the Moon. Do not do the calculations.

**7.** Refer to Table 4.1 on page 218. Calculate the true weight of a 60.0-kg astronaut on

(a) the surface of Mars, and

(b) the surface of Saturn.

**8.** Objects A and B experience a gravitational force of magnitude $2.5 \times 10^{-8}$ N. Determine the magnitude of the gravitational force if the separation distance is halved, $m_A$ increases by 8 times, and $m_B$ is reduced to $\frac{1}{4}$ of its original value.

**9.** Suppose a 65-kg astronaut on Mars is standing on a scale calibrated in newtons in an elevator. What will be the reading on the scale when the acceleration of the elevator is

(a) zero?

(b) 7.2 m/s$^2$ [up]?

(c) 3.6 m/s$^2$ [down]?

**10.** A 50-kg rock in a Nahanni River canyon breaks loose from the edge of a cliff and falls 500 m into the water below. The average air resistance is 125 N.

(a) What is the average acceleration of the rock?

(b) How long does the rock take to reach the water?

(c) What is the true weight of the rock?

(d) Does the rock have an apparent weight? Explain.

## Extensions

**11.** Research why astronauts do exercises in space and why they have difficulty walking when they return to Earth. Write a short paragraph of your findings. Begin your search at www.pearsoned.ca/school/physicssource.

**12.** Pilots in high-speed planes are subject to *g* forces. Healthy people can withstand up to 3–4 *g*s. Beyond that limit, the blood will pool in the lower half of the body and not reach the brain, causing the pilot to lose consciousness. Research how Dr. Wilbur Franks from Toronto found a solution to this problem. What connection does this problem have to human survival during a space flight? Begin your search at www.pearsoned.ca/school/physicssource.

## Consolidate Your Understanding

**13.** Write a paragraph summarizing Newton's law of gravitation. Include a numerical example that illustrates the law. Show a detailed solution.

### Think About It

Review your answers to the Think About It questions on page 195. How would you answer each question now?

### eTEST

To check your understanding of gravitation concepts, follow the eTest links at www.pearsoned.ca/school/physicssource.

# Tire Design, Stopping Distance, and Vehicle Mass

## Scenario

Imagine that you work for Alberta Infrastructure and Transportation. A level section of highway in the mountains has an abnormally large number of accidents. The surface of one lane of the highway is concrete and the other is asphalt. You are a member of a research team formed to determine the stopping distances in the summer for both wet and dry days for different types of tires and for different masses of vehicles. Your team is to prepare a written report on your findings for the Traffic Branch of the Ministry. Assume that the vehicles are travelling at the posted speed limit of 90 km/h when the brakes are applied and that the vehicles are not equipped with anti-lock braking systems (ABS). Assume that the reaction time of drivers before they apply the brakes is 1.8 s.

## Planning

Form a team of three to five members. Summarize the question your group is researching. Make hypotheses about how tire tread, surface condition, and vehicle mass might affect stopping distance. Assign roles to different team members. Some examples are team leader, materials manager, liaison officer, record keeper, and safety officer. Brainstorm strategies for researching and reporting on the question and create a timeline. Research tire designs that are designed to work well on both wet and dry road surfaces. Use the Internet and consult local tire suppliers.

## Materials

- a digital camera and a computer
- force-measuring equipment
- mass-measuring equipment
- new and used tires having different treads and designs
- vehicle brochures

## Procedure

1. Research the Internet and interview different tire suppliers to identify tires designed to work well at above-freezing temperatures on both wet and dry pavement.

**Assessing Results**

After completing the project, assess its success based on a rubric designed in class* that considers

- research strategies
- experiment techniques
- clarity and thoroughness of the written report
- effectiveness of the team's presentation
- quality and fairness of the teamwork

2. Visit a local tire supplier and borrow new and used tires of different designs. Photograph the tires to record the tread designs.
3. Design and conduct an experiment to determine the coefficients of static and kinetic friction for the tires on wet and dry asphalt, and wet and dry concrete. Recall that you will need the mass of the tires and the local value of gravitational field strength.

**CAUTION: Take all your measurements on parking lots or sidewalks, and beware of traffic. Consult the local police department and ask for supervision while doing your experiments.**

4. Research the masses of at least four small, medium, and large vehicles travelling highways that would use these tires. Determine the average mass of each class of vehicle. Sales brochures from vehicle dealers have mass information.
5. Determine the average deceleration on both wet and dry roadways for the vehicles of different mass equipped with these tires. Remember that some drivers may lock their brakes while braking.
6. Determine the stopping distance for the different vehicles under different conditions. Remember to consider driver reaction time.
7. Write a report of your findings. Use graphs and tables where appropriate. Prepare the report in a format suitable for publishing in a school magazine or local newspaper.

## Thinking Further

Write a three-paragraph addition to your team's report hypothesizing how driver education, changing the posted speed limit, and requiring that vehicles be equipped with ABS brakes might affect the results.

*Note: Your instructor will assess the project using a similar assessment rubric.

# UNIT II SUMMARY

## Unit Concepts and Skills: Quick Reference

| Concepts | Summary | Resources and Skill Building |
|---|---|---|
| **CHAPTER 3** | **Forces can change velocity.** | |
| | **3.1 The Nature of Force** | |
| Force | Force is a push or a pull on an object. Force is a vector quantity measured in newtons ($1\ N = 1\ kg{\cdot}m/s^2$). | 3-1 QuickLab<br>3-2 QuickLab<br>Examples 3.1–3.4 |
| Net force | Net force is the vector sum of two or more forces acting simultaneously on an object. A free-body diagram helps you write the net force acting on an object. | Examples 3.2–3.4 |
| | **3.2 Newton's First Law of Motion** | |
| Newton's first law | Newton's first law states that an object will continue being at rest or moving at constant speed in a straight line unless acted upon by a non-zero net force. | 3-3 QuickLab |
| | **3.3 Newton's Second Law of Motion** | |
| Newton's second law | Newton's second law states that when a non-zero net force acts on an object, the object accelerates in the direction of the net force. The magnitude of the acceleration is directly proportional to the magnitude of the net force and inversely proportional to the mass of the object. | 3-5 Inquiry Lab<br>Figures 3.33–3.35<br>3-6 Design a Lab<br>Examples 3.5–3.11 |
| | **3.4 Newton's Third Law of Motion** | |
| Newton's third law | Newton's third law states that if object A exerts a force on object B, then B exerts a force on A that is equal in magnitude and opposite in direction. | 3-7 QuickLab<br>Figures 3.53–3.57, 3.63, 3.65<br>3-8 QuickLab<br>Examples 3.12, 3.13<br>3-9 Design a Lab |
| | **3.5 Friction Affects Motion** | |
| Types of friction | Friction is a force that opposes either the motion of an object or the direction the object would be moving in if there were no friction. Static friction is present when an object is stationary but experiences an applied force. Kinetic friction is present when an object is moving. | 3-10 QuickLab<br>Figures 3.70, 3.74, 3.78–3.80<br>Examples 3.14–3.16 |
| Factors affecting friction | The magnitude of the force of friction acting on an object is directly proportional to the normal force on the object. | 3-11 Inquiry Lab |
| Coefficients of static and kinetic friction | The coefficients of friction are proportionality constants that relate the magnitude of the force of friction to the magnitude of the normal force. Temperature, moisture, and the smoothness or roughness of the contact surfaces, and the materials forming the contact surface are some factors that affect the value of the coefficients of friction. | Table 3.4<br>Examples 3.17–3.20 |
| **CHAPTER 4** | **Gravity extends throughout the universe.** | |
| | **4.1 Gravitational Forces due to Earth** | |
| Gravitational force | Gravitational force is a fundamental force, and can be described as an action-at-a-distance force or as a field. | 4-1 QuickLab |
| Gravitational field strength | Gravitational field strength is the ratio of gravitational force to mass at a specific location. The units of gravitational field strength are N/kg. | 4-2 QuickLab<br>Figure 4.10 |
| | **4.2 Newton's Law of Universal Gravitation** | |
| Newton's law of universal gravitation | Newton's law of universal gravitation states that the gravitational force of attraction between any two masses is directly proportional to the product of the masses and inversely proportional to the square of the separation distance between the centres of both masses. | Figures 4.11–4.13, 4.16, 4.24<br>Examples 4.1–4.4 |
| | **4.3 Relating Gravitational Field Strength to Gravitational Force** | |
| Calculating $g$ anywhere in the universe | Newton's law of gravitation can be used to determine the magnitude of gravitational field strength anywhere in the universe. The magnitude of gravitational field strength at a location is numerically equal to the magnitude of gravitational acceleration. | 4-3 Design a Lab<br>Examples 4.5–4.6 |
| Variations of $g$ | The value of $g$ at Earth's surface depends on latitude, altitude, the composition of Earth's crust, and Earth's rotation about its axis. | Figures 4.34, 4.35 |
| True weight, apparent weight, free fall, and weightlessness | The true weight of an object is equal to the gravitational force acting on the mass, and depends on location. Apparent weight is the negative of the normal force acting on an object. Free fall is the condition where the only force acting on an object is the gravitational force. True weightlessness is the condition in which $\vec{w} = 0$ for an object and $\vec{F}_g = 0$ on the object. | Figures 4.36–4.40, 4.43, 4.44, 4.46<br>4-4 QuickLab<br>Examples 4.7, 4.8 |

## Vocabulary

**1.** Using your own words, define these terms, concepts, principles, or laws.
action-at-a-distance force
action force
apparent weight
coefficient of friction
field
free-body diagram
free fall
gravitational field strength
gravitational force
gravitational mass
inertia
inertial mass
kinetic friction
net force
Newton's first law
Newton's law of gravitation
Newton's second law
Newton's third law
normal force
reaction force
static friction
tension
true weight

## Knowledge

### CHAPTER 3

**2.** An object experiences three forces: $\vec{F}_1$ is 60 N [22.0°], $\vec{F}_2$ is 36 N [110°], and $\vec{F}_3$ is 83 N [300°]. Explain, using words and diagrams, how to calculate the net force on the object. What is the net force?

**3.** An object experiences zero net force. Work with a partner to describe the possibilities for its motion.

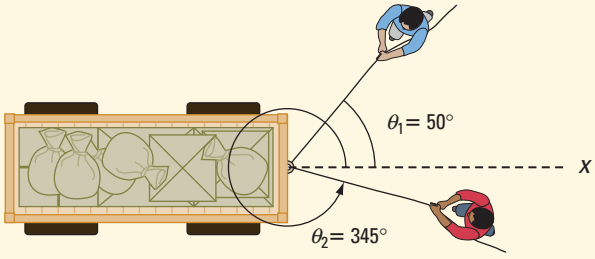
**4.** A person with a plaster cast on an arm or leg experiences extra fatigue. Use Newton's laws to explain to a classmate the reason for this fatigue.

**5.** Use inertia and Newton's first law to explain how the spin cycle in a washing machine removes water from wet clothes.

**6.** A load is placed on a 1.5-kg cart. A force of 6.0 N [left] causes the cart and its load to have an acceleration of 3.0 m/s² [left]. What is the inertial mass of the load?

**7.** What happens to the acceleration of an object if the mass is constant and the net force
(a) quadruples?
(b) is divided by 4?
(c) becomes zero?

**8.** Two people, A and B, are pulling a wagon on a horizontal, frictionless surface with two ropes. Person A applies a force [50°] on one rope. Person B applies a force of 25.0 N [345°] on the other rope. If the net force on the wagon is 55.4 N [26°], calculate the magnitude of person A's applied force.




**9.** A book is at rest on a table. The table is exerting an upward force on the book that is equal in magnitude to the downward force exerted by the book on the table. What law does this example illustrate?

**10.** A pencil exerts a force of 15 N [down] on a notebook. What is the reaction force? What object is exerting the reaction force?

**11.** Explain why the coefficients of static and kinetic friction are numerals without units.

**12.** Draw a free-body diagram for a stationary 5.0-kg block resting on a rough incline forming an angle of 30.0° with the horizontal.
(a) Explain why the block is stationary.
(b) Explain why a free-body diagram is helpful to describe the situation.

**13.** How does the ability of a car slowing down on wet asphalt compare to it slowing down on wet concrete? Refer to Table 3.4 on page 183.

**14.** Compare the acceleration due to gravity and the gravitational field strength at the top of a tall skyscraper on Earth.

**15.** Consider the quantities gravitational force, mass, and gravitational field strength. Which of these quantities affects the inertia of an object?

**16.** Suppose an athlete were competing in the 2010 Winter Olympics in Vancouver and Whistler, British Columbia. Whistler has an elevation of 2182 m at the top and 652 m at the base. If the ski jumping and bobsled events are held near the top of the mountain rather than at the base, how might the results of these events be affected?

**17.** Two bags, each containing 10 oranges of equal mass, are hung 4 m apart. In a small group, describe two situations, one involving mass and the other involving separation distance, that would double the gravitational force exerted by one bag on the other. Explain your answer.

**18.** A student working on a satellite problem got an answer of $57.3\ \frac{N \cdot s^2}{m}$. What physical quantity was the student solving for?

**19.** Use an example to explain the meaning of the statement: "The gravitational force exerted by Mars on a space probe varies inversely as the square of the separation distance between the centre of Mars and the centre of the probe."

**20.** Is an object in free fall weightless? Explain your reasoning.

**21.** Compare the gravitational force exerted by Earth (mass $M$) on two satellites (masses $m$ and $2m$) in orbit the same distance from Earth.

**22.** Compare the magnitude of Earth's gravitational field strength at the equator and at the North Pole. Explain your answer to a classmate.

**23.** On Earth, how does the mass of an object affect the values of the quantities below? Explain your answers.

(a) acceleration due to gravity

(b) gravitational field strength

## Applications

**24.** Two horizontal forces act on a soccer player: 150 N [40.0°] and 220 N [330°]. Calculate the net force on the player.

**25.** Calculate the acceleration of a 1478-kg car if it experiences a net force of 3100 N [W].

**26.** A car is stopped at a stoplight facing due east. When the light turns green, the car gradually speeds up from rest to the city speed limit, and cruises at the speed limit for a while. It then enters a highway on-ramp and gradually speeds up to the highway speed limit all the while heading due east. Sketch a free-body diagram for the car during each stage of its motion (five diagrams in total).

**27.** A net force of magnitude 8.0 N acts on a 4.0-kg object, causing the velocity of the object to change from 10 m/s [right] to 18 m/s [right]. For how long was the force applied?

**28.** Two people, on opposite banks, are towing a boat down a narrow river. Each person exerts a force of 65.0 N at an angle of 30.0° to the bank. A force of friction of magnitude 104 N acts on the boat.

(a) Draw a free-body diagram showing the horizontal forces acting on the boat.

(b) Calculate the net force on the boat.

**29.** A force acting on train A causes it to have an acceleration of magnitude 0.40 m/s$^2$. Train A has six cars with a total mass of $3.0 \times 10^5$ kg, and a locomotive of mass $5.0 \times 10^4$ kg. Train B has a locomotive of the same mass as train A, and four cars with a total mass of $2.0 \times 10^5$ kg. If the same force acts on train B, what will be its acceleration? Ignore friction.

**30.** A submarine rescue chamber has a mass of 8.2 t and safely descends at a constant velocity of 10 cm/s [down]. If $g = 9.81$ m/s$^2$, what is the upward force exerted by the cable and water on the chamber?

**31.** A 240-kg motorcycle and 70-kg rider are travelling on a horizontal road. The air resistance acting on the rider-bike system is 1280 N. The road exerts a force of static friction on the bike of 1950 N [forward]. What is the acceleration of the system?

**32.** The velocity of a 0.25-kg model rocket changes from 15 m/s [up] to 40 m/s [up] in 0.60 s. Calculate the force that the escaping gas exerts on the rocket.

**33.** Two boxes, A and B, are in contact and initially stationary on a horizontal, frictionless surface. Box A has a mass of 60 kg and box B a mass of 90 kg. A force of 800 N [right] acts on box A causing it to push on box B.

(a) What is the acceleration of both boxes?

(b) What is the magnitude of the action-reaction forces between the boxes?

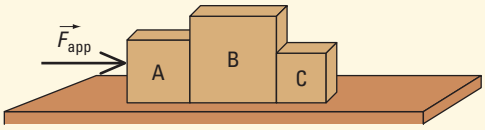
**34.** A person exerts a force of 1.5 N [right] to pull a 2.0-kg block of glass at constant velocity along a horizontal surface on the Moon ($g_{Moon}$ = 1.62 m/s$^2$). What is the coefficient of kinetic friction for the glass on the surface?

**35.** Three oak blocks, $m_A$ = 4.0 kg, $m_B$ = 6.0 kg, and $m_C$ = 3.0 kg, are positioned next to each other on a dry, horizontal oak surface. Refer to Table 3.4 on page 183.

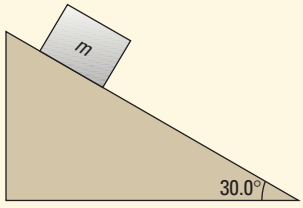
(a) What horizontal force must be applied to accelerate the boxes at 1.4 m/s$^2$ [forward]?

(b) Calculate the force exerted by $m_B$ on $m_C$.

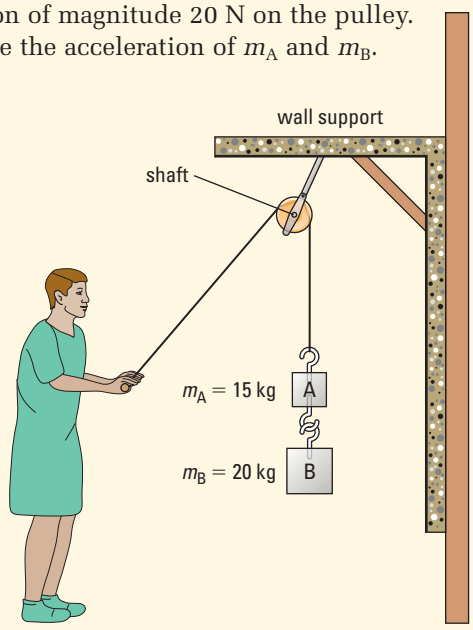
(c) Calculate the force exerted by $m_B$ on $m_A$.




**36.** A 10.0-kg block is placed on an incline forming an angle of 30.0° with the horizontal. Calculate the acceleration of the block if the coefficient of kinetic friction for the block on the incline is 0.20.




**37.** A rehabilitation clinic has a device consisting of a light, frictionless pulley attached to a wall support. A patient pulls with a force of magnitude 416 N. The rope exerts a force of friction of magnitude 20 N on the pulley. Calculate the acceleration of $m_A$ and $m_B$.



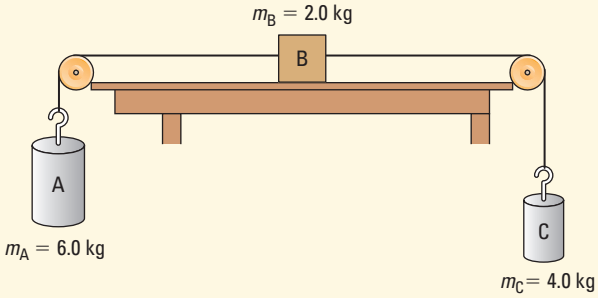

**38.** Three objects, A, B, and C, are connected together by light strings that pass over light, frictionless pulleys. The coefficient of kinetic friction for object B on the surface is 0.200.

(a) What is the acceleration of object B?

(b) What is the tension in each string? Explain why the tensions are different.

(c) Draw a free-body diagram for object B.

(d) Identify four action-reaction pairs associated with object B.




**39.** The gravitational force on an object located $2r_{Earth}$ from Earth's centre is 200 N [toward Earth's centre]. What is the gravitational force if the object is $10r_{Earth}$ from Earth's centre?

**40.** A 50-kg diver steps off a 9.0-m diving tower at the same time as a 100-kg diver. Work with a partner to compare the times taken for the two divers to reach the water. Ignore air resistance.

**41.** Skylab 1, the first American space station, had a mass of about 68 t. It was launched into orbit 435 km above Earth's surface. Calculate the gravitational field strength at the location of Skylab 1 at this altitude. Refer to Table 4.1 on page 218.

**42.** A spring scale is used to measure the gravitational force acting on a 4.00-kg silver block on Earth's surface. If the block and spring scale are taken to the surface of Mars, by how much does the reading on the spring scale change? Refer to Table 4.1 on page 218.

**43.** A 60-kg student is standing on a scale in an elevator on Earth. What will be the reading on the scale when the elevator is

(a) (i) moving down at constant speed?

(ii) at rest at a floor?

(iii) accelerating at 4.9 m/s$^2$ [up]?

(iv) accelerating at 3.3 m/s$^2$ [down]?

(b) What is the student's apparent weight and true weight in all the situations in part (a)?

**44.** A 60-kg skydiver falls toward Earth with an unopened parachute. The air resistance acting on the skydiver is 200 N [up]. What is the true weight and acceleration of the skydiver?

**45.** A group of tourists on a ledge overlooking Pulpit Rock, Northwest Territories, dislodge a 25-kg boulder. The rock takes 8.0 s to fall 300 m into the water below. At this location, the gravitational field strength is 9.81 N/kg [down].

(a) Calculate the acceleration of the boulder.

(b) Calculate the air resistance acting on the boulder.

(c) What was the apparent weight of the boulder during its fall?

## Extensions

**46.** The value of $g$ on the Moon is less than that on Earth. So a pendulum on the Moon swings slower than it would on Earth. Suppose a pendulum is 36 cm long. Use the equation $T = 2\pi \frac{\sqrt{l}}{\sqrt{g}}$ to calculate the period, $T$, at the equator on Earth and on the Moon. Refer to Table 4.1 on page 218.

**47.** During the last seconds of a hockey game, the losing team replaces their goalie with a good shooter. The other team shoots the 150-g puck with an initial speed of 7.0 m/s directly toward the unguarded net from a distance of 32 m. The coefficient of kinetic friction for the puck on the ice is 0.08.

(a) What is the force of kinetic friction acting on the puck?

(b) What is the acceleration of the puck?

(c) How long does it take the puck to stop?

(d) Will the puck reach the net if no other player touches it?

**48.** Construct a gathering grid to distinguish among Newton's three laws. In the left column, identify the criteria you will use to compare and contrast the laws. Add three additional columns, one for each law. Then place checkmarks in the appropriate columns to compare the laws.

**49.** During the 2000 Sydney Olympics, some swimmers wore special swimsuits designed to reduce water resistance. Compare the arguments that people might make to defend or oppose the standardization of athletic equipment in the interests of fair play.

**50.** List two different stakeholders in the airbag debate and describe how their positions on the issue compare.

**51.** The G rocket of the former Soviet Union has a mass of about $3.8 \times 10^6$ kg and its first-stage engines exert a thrust of about $5.0 \times 10^7$ N [forward].

(a) What is the true weight of the rocket on Earth's surface?

(b) Calculate the net force acting on the rocket at liftoff.

(c) Calculate the initial acceleration of the rocket.

(d) What should happen to the acceleration if the force exerted by the engine remains constant as the fuel burns?

(e) Why is the first stage jettisoned after the fuel is consumed?

**52.** People who believe in astrology claim that distant planets affect the forces acting on people on Earth. In a small group, answer the questions below.

(a) Suppose the mass of the person sitting next to you is 70 kg and the separation distance between you and that person is 1.0 m. The mass of Mars is $6.42 \times 10^{23}$ kg and the separation distance between Mars and Earth is $2.3 \times 10^{11}$ m. Compare the gravitational force exerted by Mars on you and the gravitational force exerted by the person sitting next to you on you.

(b) Comment on the claim made by astrologers based on your answer to part (a).

**53.** In a small group, research the materials being used to make artificial joints such as hips and knees. Find out how they are designed to provide enough friction for stability but not so much friction that the joints cannot move. Begin your search at www.pearsoned.ca/school/physicssource.

**54.** Manufacturers of skis recommend different waxes for different snow temperatures. Design and carry out an experiment to test the recommendations for three different waxes.

**55.** Research gait analysis, the study of how humans walk and run. This topic is central to physiotherapy, orthopedics, the design of artificial joints and sports footwear, and the manufacture of orthotics. How do Newton's three laws apply to gait analysis? Interview people associated with rehabilitation and sports. Write a brief summary of your findings. Begin your search at www.pearsoned.ca/school/physicssource.

**eTEST**

To check your understanding of dynamics, follow the eTest links at www.pearsoned.ca/school/physicssource.